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Pollution Taxation and Environmental Quality

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1 Introduction

The use of pollution taxation has become very popular in recent decades for policy making. According to the basic public finance theory, the optimal pollution tax in the height of the marginal social damage of pollution, which is also called Pigouvian tax, corrects a negative external effect perfectly and leads therefore to a desirable environmental quality.

Introducing the existence of distortionary taxes into this partial equilibrium consideration, it could be supposed that increasing the pollution tax increases the environmental quality. Because the consumption of the pollution producing commodity is reduced and at the same time the generated revenues can be used to cut other distortionary taxes which is known as the double-dividend hypothesis. An optimised taxation system would lead therefore to a pollution tax rate which exceeds the Pigouvian level, if the marginal revenue of the pollution tax is positive as in the discussion assumed. Because other markets are affected as well, the connection between optimal pollution taxation and the environmental quality has to be determined in a general equilibrium model which is the focus of this thesis.

In the first-best treatment of a general equilibrium model, where lump-sum taxation is feasible, the Pigouvian tax holds as optimal. If distortionary taxes are necessary to finance public spending, the optimal pollution tax can either exceed or fall short of the Pigouvian level. Different tax combinations lead to the same optimal allocation in this so-called second-best treatment where the excess burden of the taxation system is minimised. Hence, the conclusion of the partial equilibrium model that a change of the tax rate automatically has an impact on the environmental quality can not hold. Therefore, the reached allocations instead of the optimal prices have to be viewed for determining the effects of the entire tax system on the environmental quality.

It can be shown that the second-best taxation system leads to a higher environmental quality compared to the first-best, in contrary to the provision of the public good which will be reduced. The use of pollution taxation leads to a different treatment of these two, although they have mainly the same properties. While public spending becomes more costly if distortionary taxes are used instead of lump-sum taxation, the by the pollution taxation generated revenues are more valuable. Hence, the existence of tax distortions has a positive impact on the environment.

Additional government spending and therefore an increase in tax distortions
will not - different from the double-dividend expected - favour pollution taxes as source of revenue. It can be shown that the environmental component of the optimal pollution tax decreases in this case. While it could be assumed that the environmental quality falls as well, an increase in tax distortions will influence the environmental quality by two different effects. On the one hand, environmental quality is negatively affected by a commodity substitution effect, which let individuals consume more polluting commodities, and on the other hand, a leisure substitution effect reduces overall consumption and has therefore a positive influence on the environment. For reasonable parameter values the leisure substitution effect is stronger and therefore a higher environmental quality will be reached by the use of distortionary taxes.

Due to these considerations it can be concluded that a pollution tax in a tax system where the deadweight loss is minimised will lead to a desirable improvement of the environment. Policy making can therefore set the focus on optimising the taxation system. However, if changes in the tax system occur, the impact on the environment can not be deduced solely from knowing the change in the pollution tax rate.

The structure of the thesis is as follows. Chapter 2 shows the importance of pollution taxes in practice and gives a summary of the partial equilibrium analysis. Chapter 3 discusses the general equilibrium model using the papers of Fullerton (1997), Gaube (2005) and Metcalf (2003). Chapter 4 concludes.
2 Approach

2.1 Relevance of Pollution Taxation

A policy instrument that is used for reaching environmental targets has to change the behaviour of consumption or production to be suitable. Pollution taxation influences the consumer’s decision through an increase in prices of the polluting commodity. For reaching environmental targets the use of pollution taxation became quite popular however the advantages and disadvantages of the use of taxation compared to other policy instruments are described in literature.

In the 1970s and 1980s nearly the whole environmental policy was based on regulations. During the 1980s the interest in market-based instruments started. Between 1987 and 1994 alone, in the OECD countries the number of environmentally related taxes and charges increased over 50 per cent. (Ekins, 1999)

Today pollution taxes\(^1\) are implemented in many different goods for example taxes on fuels, tires, mining, pesticides, waste, water quantity or plastic bags. The OECD (2006) declares the existence of more than 350 environmentally related taxes in OECD countries. The most common environmental taxes are levied on energy products (150) and motor vehicles (125). In addition there can be found 50 waste-related taxes within OECD countries. The remaining taxes are levied on a broad spectrum of different tax-bases. These taxes generate revenues on average between 2% and 2.5% of the GDP of the observed countries, which is on average between 6% and 7% of the total tax revenue.

Because the gained share of public revenue does not seem very high, one might think that the importance of pollution taxes within policy making is over-estimated. As already mentioned polluting behaviour should be reduced by pollution taxation and therefore the generated revenues of one period can not be used as an indicator for effectiveness because the research and development sector as well as the individuals will need time to adapt their behaviour. In a long-term context this development can be seen as an argument against the use of pollution taxation for financing government spending. The OECD (2006, 50f) emphasizes that short observation periods and the fact that pollution taxes are mostly used combined with other policy instruments makes it difficult to define exactly the effect of certain taxes for empirical research.

\(^1\)The terminology of pollution tax is within policy making often used in a slightly different way, namely for taxes on measured or estimated emissions. In politics the terms environmental taxes or environmental related taxes are more common. In this thesis the terminology pollution tax is used in the meaning of economic theory and includes therefore all taxes that are set to influence the market due to environmental issues.
However it should be mentioned that pollution taxation plays a role in financing the government budget even if the proportion is not that high. Hence, one can conclude that environmental concerns will probably not always be the reason for implementing pollution taxes.²

2.2 Partial Equilibrium Analysis

Although environmental quality is non-rival and non-excludable in consumption and therefore possesses the properties of a public good, its economic treatment differs in the discussed model. The reasons and results of these differences will be discussed in chapter 3.3.

In contrary to public goods, the decision of the environmental quality is not steered by policy makers but instead by the general population. The quality of environment can be seen as a basic endowment and the reached level is determined by the consumption decision of the individuals which do not take the by the commodities caused pollution into account. Apparently, there exists a difference between the optimal decision of individuals and an optimal decision for society. Therefore, pollution can be seen as a negative external effect.

For an efficient market the government can force the individuals to take the occurring external cost into account and consequently correct the market failure. Due to this, the base of environmental policy can be found in the economic theory of externalities as seen in every public finance and environmental policy textbook.

In the case of negative external effects such as pollution, marginal social cost exceeds private marginal cost so, too much of the polluting commodity is consumed. Hence, the government can set a tax at the height of the cost difference - the marginal social damage - on the consumption of the pollution producing commodity. As a consequence, individuals value the costs of the polluting good correctly and the market works efficiently, which is defined as the equilibrium at which the cost of marginal social damage is equal to the cost of abatement, i.e. the pollution tax. This optimal pollution tax, which implements the externalities, is called Pigouvian tax, and does not lead to any deadweight loss. Within a partial equilibrium model, where only the market of the pollution producing commodity is observed, the effects of the pollution tax on the environmental quality are easy to determine. An increase of the pollution tax will lead to less consumption of the dirty commodity and therefore to a higher environmental quality.

Based on the partial equilibrium model the origin of the so called double-

²(Oates, 1993, 135) designates as an example, that in 1987 the United States implemented a tax on sulfur and nitrogen oxide emissions only to reduce the deficit in the federal budget.
dividend hypothesis\textsuperscript{3} can be found which claims that - if other, distortionary taxes exist in an economy - a pollution tax does not only improve the environmental quality as a first dividend, but using the generated revenues to cut other distortionary taxes leads to a second dividend.

If an increase in the pollution tax rate leads to an increase in tax revenue as assumed in further discussion, to set the pollution tax above the Pigouvian level has to be the optimal tax policy within this framework. (Oates, 1993) At first sight this statement seems to be plausible but in the following will be shown that this conclusion can not hold within a general equilibrium model, where the effects of other markets are taken into account.

\textsuperscript{3}For a survey of the double dividend-hypothesis and discussion of different definitions see for example, Goulder (1994) or Schöb (2003).
3 General Equilibrium Model

3.1 Description of the Model

The used general equilibrium model which is presented in this chapter is the base for among others, the discussed papers of Fullerton (1997), Gaube (2005) and Metcalf (2003), as well as for the cited paper of Bovenberg and de Mooij (1994).

In the observed economy $N$ identical individuals exist. These decide the level of the consumption of a clean commodity $C$, a dirty commodity $D$ and leisure $V$. Further, the public good $G$ and the environmental quality $E$ influence the utility of the individuals. The strictly quasiconcave utility function $U(C, D, V, E, G)$ is monotonically increasing in the mentioned arguments.

Every individual has an endowment of time that is normalised to one in this model. Hence, labour $L$ can also be expressed as $(1 - V)$ and is used to produce the commodities $C$, $D$, and $G$. The rates of transformation between these commodities are normalised to unity. The constant productivity of labour is defined as $h$. Therefore, the production frontier of the economy is

$$NhL - NC - ND - G = 0.$$  \hspace{1cm} (1)

The consumption of $D$ produces a negative external effect on the environment. Hence, the total amount of the consumed dirty commodity $ND$ affects the environmental quality $E$ which can be written as

$$E = e(ND), \quad e'(ND) < 0.$$  \hspace{1cm} (2)

The consumers take the prices $p_C = 1 + t_C$, $p_D = 1 + t_D$ and $p_L = h(1 - t_L)$, where the subscripts refer to the related commodities, as well as the quantities of $G$ and $E$ as given. The individual’s budget constraint is therefore

$$p_L(1 - V) - p_C C - p_D D = 0.$$  \hspace{1cm} (3)

Individuals do not take the negative externalities of the consumption of the dirty commodity $D$ into account, when they maximise their utility with respect to $C$, $D$ and $V$

$$\max_{C,D,V} U(C, D, V, E, G) \quad s.t. \quad (3).$$  \hspace{1cm} (4)

This leads to the demand functions for $C(p_C, p_D, p_L, E, G)$, $D(p_C, p_D, p_L, E, G)$ and $V(p_C, p_D, p_L, E, G)$, which depend on the prices of the commodities and the
amounts of $E$ and $G$. Due to these an indirect utility function can be written as $W(p_C, p_D, p_L, E, G)$.

The first-best maximisation problem

$$\max_{C,D,V,E,G} U(C, D, V, E, G) \quad \text{s.t.} \quad (1) \quad \text{and} \quad (2),$$

(5)
describes the potential of the economy. Its corresponding optimal allocation will be written as $(C^F, D^F, V^F, E^F, G^F)$. The first-order conditions of (5) imply the Samuelson conditions, namely

$$\frac{\partial U}{\partial D} + N \frac{\partial U}{\partial E} \epsilon' = \frac{1}{h}, \quad \frac{\partial U}{\partial V} + N \frac{\partial U}{\partial E} \epsilon' = 1,$$

(6)
for $E$ and

$$\frac{N \partial U}{\partial V} = \frac{1}{h}, \quad \frac{N \partial U}{\partial C} = 1,$$

(7)
for $G$.

By comparing the first-order conditions of (4) and (5) it can be seen easily that the efficiency conditions are satisfied if the consumer price of the dirty commodity is corrected by a tax, namely

$$\tau = - \frac{N \frac{\partial U}{\partial E} \epsilon'}{\lambda} = t_D.$$

(8)
This term is exactly the marginal social damage of pollution which is the marginal damage that an extra unit of consumption of the dirty commodity $D$ has on all $N$ individuals. It is converted into monetary terms by the division of $\lambda$, the marginal utility of income as seen in the first-order conditions. Hence, the efficient allocation can be reached if the tax on the pollution producing commodity equals the Pigouvian tax.

This first-best pollution tax will hold if the unlikely coincidence occurs that the amount of revenue that is generated by the Pigouvian tax is sufficient to cover the public spending or if lump-sum taxation is feasible. Because it is unrealistic that neither of these cases will happen, distortionary taxes are necessary to finance the government spending.

If different tax rates are implemented into the model, the government maximises welfare - which is defined as the utility of the identical individuals in this model - by choosing the tax rates $t_C, t_D, t_L$ and the level of the provided public good $G$ subject to the environmental externalities, and the government’s budget
which guarantees the financing of the public good. This second-best maximisation problem can be written as

$$
\max_{t_C, t_D, t_L, G} W(p_C, p_D, p_L, E, G) \quad \text{s.t.} \quad (2) \quad \text{and} \quad (9),
$$

where \((C^S, D^S, V^S, E^S, G^S)\) is the corresponding second-best allocation. At this point it should be noted that maximising the welfare is equivalent to minimising the dead weight loss of a tax system.

### 3.2 The Height of the Tax Rate

The first intuitive thought about the quality of the environment in an economy that uses pollution taxation as policy instrument, leads to the height of the tax rate. One might think that with a higher tax rate on the dirty commodity, less pollution automatically occurs as has been shown as well by the results of the partial equilibrium model.

As an answer on the partial equilibrium discussion and its conclusions Bovenberg and de Mooij (1994) discussed the optimal pollution tax rate in the already mentioned general equilibrium model. In this important contribution they deal with an economy that uses distortionary taxes on labour and no taxation on the clean commodity. They demonstrated that the optimal second-best pollution tax\(^4\) lies below the Pigouvian tax, which could lead to the misinterpretation that the environmental quality falls as well short of the optimal Pigouvian level.

It is easy to show within the presented model that the tax rate on the dirty commodity depends only on the normalisation that is used for calculating the second-best optimum and can therefore exceed or fall short of the Pigouvian tax. For the following treatment the paper of Fullerton (1997) is used.

#### 3.2.1 Normalisation

For explaining the effects on the optimal pollution tax rates, different possibilities to finance the provision of the public good are viewed which is seen as constant \((dG = 0)\).

\(^4\)Another interpretation could be that they discuss the difference between the tax rates \(t_D\) and \(t_C\), and therefore the environmental component of the optimal pollution tax as in chapter 3.4 which would be the same because of the used normalisation.
The total differential of the utility function $U(C, D, V, E, G)$ is set to zero because no utility improvement in the second-best optimum can be invented. Using the properties $dG = 0$ and $dV = -dL$ one can write

$$dU = 0 = -\frac{\partial U}{\partial V} dL + \frac{\partial U}{\partial C} dC + \frac{\partial U}{\partial D} dD + \frac{\partial U}{\partial E} e' N dD.$$ (11)

After inserting the first-order conditions of the individual's maximisation problem (see appendix A.1), equation (8) and using the differentiated production frontier (1), namely $hdL = dC + dD + \frac{dG}{N}$ where $dG=0$, one gets

$$0 = h t_L dL + t_C dC + (t_D - \tau) dD.$$ (12)

If the tax on labour as well as the tax on the clean commodity are set to zero ($t_L = t_C = 0$) the Pigouvian tax as optimal solution holds. This case equals the already presented first-best setting.

In the first normalisation that is treated $t_C = 0$ and $t_L > 0$ are assumed which means that the government spending is financed by a labour tax, while the market for the clean commodity stays tax-free. Inserting these assumptions into equation (12) leads to the optimal condition of

$$t_D - \tau = -h t_L \frac{dL}{dD}.$$ (13)

The decisive term $\frac{dL}{dD}$ is positive which is the already mentioned result of the paper of Bovenberg and de Mooij (1994). Therefore, it is shown that the second-best pollution tax falls short of the first-best Pigouvian tax ($t_D^S < \tau$).

The second normalisation assumes that no labour tax is used ($t_L = 0$). Thus, additional public revenue is generated by a tax on the clean commodity ($t_C > 0$). Resultingly equation (12) leads to

$$t_D - \tau = -t_C \frac{dC}{dD}.$$ (14)

Because the labour market is not affected, and revenue neutrality is assumed, an increase of $t_D$ implies a decrease of $t_C$. Hence the term $\frac{dC}{dD}$ has to be negative. It is clearly seen that this second-best pollution tax exceeds the Pigouvian level ($t_D^S > \tau$).

Fullerton (1997, 249) also shows that besides the above treated normalisations the tax on dirty commodities can be set to zero ($t_D = 0$). Therefore, the taxes on clean commodities and labour are used to finance the government spending.
In this treatment a part of the raised labour tax is given back as a subsidy on the clean commodity. This tax combination leads to the same second-best allocation as the others. Thus the same environmental quality can be reached without using any tax on the pollution producing commodity.

As shown above three tax combinations lead to different tax rates but to the same optimal allocations. The used normalisation does not have any effect on the second-best allocation and therefore nor on the optimal amount of the supplied public good or the environmental quality. Consequently, no conclusion from these results on the environmental quality can be drawn, but a change in relative prices can be observed. Because of the tax rate change compared to the first-best, the marginal rates of substitution as known from the Samuelson conditions (6) and (7) change.\(^5\)

Therefore, \( t_D^S < \tau \) leads to

\[
\frac{\partial U}{\partial D} + \frac{Ne'}{\partial U} e' < 1, \quad (15)
\]

which shows that the tax differential between the tax rates in second-best is lower than in first-best. This means the dirty commodity becomes proportionally cheaper compared to the clean commodity in second-best.

For \( t_D^S > \tau \) the tax differential between the dirty commodity and leisure is higher in the second-best than in the first-best treatment which is seen in

\[
\frac{\partial U}{\partial D} + Ne' \frac{\partial U}{\partial V} > \frac{1}{h}, \quad (16)
\]

Hence, the dirty commodity is proportionally more expensive to leisure in second-best.

### 3.2.2 Discussion of the Results

As it has been shown the second-best allocation can be reached by different tax combinations. One tax normalisation leads to an optimal pollution tax that is lower than the Pigouvian level, whereas another optimal pollution tax can be found above the Pigouvian tax and a third normalisation does not have a tax rate on the pollution producing commodity at all.

The connection between the tax normalisation and the resulting height of the pollution tax rates can be explained intuitively. Within this discussion the labour

\(^5\)The mathematical treatment originates from Gaube (2005).
tax can be seen as a uniform commodity tax. Therefore, it can be replaced by raising the taxes $t_C$ and $t_D$ simultaneously. Hence, revenue is solely generated by the used labour tax in the first normalisation. Because the environmental component of the optimal pollution tax is reduced in a distortionary taxation system as will be seen as well in chapter 3.4, it is obvious that the optimal pollution tax falls short of the Pigouvian level if solely labour taxation is used.

These differences demonstrate that the knowledge if the second-best pollution tax exceeds or falls short of the Pigouvian tax is not sufficient to give any statement about its impacts on the environment and the public good. The reached allocations of an optimised taxation system are not affected by the used normalisation and therefore the tax combination.

These calculations offer no conclusion to the question if distortionary taxes influence the environmental quality. If besides a desirable amount of environmental quality can be reached within a second-best optimal taxation system, can not be answered either. For comparing first-best and second-best environmental quality there is no way around having a look at the allocations instead of the tax rates and therefore the quantities instead of the prices.

In reality it is obvious that the tax system for financing government expenditure consists of many different taxes. These implements that to try to explain the real financing system in the model would mean that all commodities are taxed ($t_C \neq 0$, $t_D \neq 0$ and $t_L \neq 0$) as seen in equation (12). Using all taxes in the model would make at least one of them redundant and can be expressed through a combination of the others as mentioned, but the change of the relative prices due to the different normalisations can give policy making a clue to find the optimal pollution tax.

Fullerton (1997, 249) argues that if economies have higher labour taxes the normalisation where the tax on the clean commodity is set to zero is more relevant, whereas if commodity taxes are the main source of revenues, the other theoretical treatment is the more interesting one. He also mentions that some policy instruments are easier to implement than others and therefore the knowledge of setting different tax combinations is an advantage. These arguments seem quite plausible, but the question about the reached environmental quality remains unanswered.

3.3 Comparison of First-Best and Second-Best Allocations

Following their calculation, Bovenberg and de Mooij (1994, 1088) argued that not only the public good consumption, but as well the environmental quality is
crowded out by high costs of public funds. One can imagine that the crowding-out argument is based on the fact that generating government revenue is more costly in the second-best optimum than in the first-best one, because of the tax distortions which generate an excess burden.

A conclusion of the mentioned crowding-out idea could be that higher spending on the public good leads automatically to less environmental quality in a second-best world, if the government spending will be kept on a constant level as it is assumed. This would mean the government has the possibility to increase environmental quality at cost to the public good and vice versa, which would lead to an inefficient allocation.

As it has been shown that solely the knowledge of the height of the optimal tax rates is not sufficient to answer the impact of pollution taxes on the environmental quality. In this chapter, instead of the prices the focus will be changed to the optimal tax systems reached allocations. Therefore, it will be investigated how the entire tax system affects the environmental quality. For the comparison of the first-best and second-best allocations the treatment of Gaube (2005) will be used.

The for this calculation used utility function, with a subutility function for the goods $C$ and $D$ can be written as

$$U(C,D,V,E,G) = M(C,D) + V + B(G) + H(E), \quad (17)$$

where $M(C,D)$, $B(G)$ and $H(E)$ are strictly concave and $e''(ND) = 0$ is assumed. Due to the additive separability of the utility, the demand functions for the commodities $C$ and $D$ are independent of $G$ and $E$. Through these strict assumptions the difference of the environmental quality $E$ and the public good $G$ can be easier expressed.

The normalisation $t_L = 0$ will be used within the treatment and hence the first-order conditions of the second-best maximisation problem (10) with respect to $t_C$, $t_D$ and $G$ can be written as

$$t_C : \quad -\frac{C}{h} + \frac{\partial U}{\partial E} Ne' \frac{\partial D}{\partial t_C} + \mu \frac{\partial R}{\partial t_C} = 0 \quad (18)$$

$$t_D : \quad -\frac{D}{h} + \frac{\partial U}{\partial E} Ne' \frac{\partial D}{\partial t_D} + \mu \frac{\partial R}{\partial t_D} = 0 \quad (19)$$

$$G : \quad \frac{\partial U}{\partial G} - \frac{\mu}{N} = 0, \quad (20)$$

where $\mu$ is the Lagrange multiplier of the governments budget constraint and $R := t_CC + t_DD + ht_LL$, the overall government tax revenue.
The first order-condition due to $t_D$ is in the following, the point of interest if the focus is set on the pollution tax and its effects. Thus, implementing the first-best allocation by means of its prices, equation (19) can be rearranged (see appendix A.3) to

$$\frac{p_S^c - p_F^c}{h} \frac{\partial D}{\partial t_C} + \left[ \frac{p_S^D - p_F^D}{h} - Ne' \left( \frac{\partial U}{\partial E^F} - \frac{\partial U}{\partial E^S} \right) \right] \frac{\partial D}{\partial t_D} + \left( \mu - \frac{1}{h} \right) \frac{\partial R}{\partial t_D} = 0,$$

(21)

Now, equations (18), (20) and (21) can be used for an comparison of the optimal allocations of the first-best and second-best allocation of $G$ and $E$.

3.3.1 Model without Cross-Price Effects

Consumers see an tax on a commodity as an increase in its price. If the dirty commodity becomes proportionally more expensive compared to others, its consumption will be reduced. Therefore, the own-price elasticity of a commodity is an important concept for the treatment and the observation of the effectiveness of pollution taxes in empirical research. If a tax increase does not change the demand, it can not be seen as successful from an environmental point of view. Such examples can be found in environmental policy.\(^6\)

Within empirical research a difference in price elasticities between short run and long run can be observed. If the focus is set for example on the energy or transport sector where pollution taxes are quite common, it is obvious that the source for energy cannot be substituted that easily in the short run and will therefore have a low own-price elasticity. It can be imagined that in long run more efficient energy production or alternative technologies are feasible and affect therefore the environmental quality.\(^7\) Anyway, these adjustments are of course not observable within the discussed model. They can only give a clue as to the reachable equilibrium.

While the importance of own-price elasticity is quite obvious, the following first theoretical consideration of the model denies the existence of cross-price effects between the commodities $C$ and $D$. The assumption that a tax change of one good does not have any influence on the consumed amount of the other - in the model the tax of the clean good does not influence the consumed amount of the dirty one and vice versa - seems to be quite strict. On the other hand, within this framework some explanations could be constructed. If the dirty commodity is

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\(^6\)e.g. The introduction of a SO2 tax in France or pesticide tax in Sweden did not have any influence on the consumption level. (OECD, 2006, 63)

\(^7\)For an overview of short run and long run price elasticities of different pollution taxes see OECD (2006, Ch.3).
energy and the clean one food for example, the absence of a cross-price elasticity does not seem that unrealistic.

Of course these assumptions simplify the calculations and the result will give a first hint of the effects even if they are attackable. In any case, in a second step the model can be extended.

As was already mentioned the provision of a public good \( G \) will be reduced if a change of the tax system from a first-best to a second-best one takes place \( (G^S < G^F) \).

A distortionary tax \( t^S_C > 0 \) on the clean commodity is assumed. The above mentioned subutility function implies because of its strict concavity that the own-price elasticities are negative, namely \( \frac{\partial C}{\partial t} < 0 \) and \( \frac{\partial D}{\partial t} < 0 \). The absence of cross-price effects can be written as \( \frac{\partial C}{\partial D} = \frac{\partial D}{\partial C} = 0 \).

These assumptions reduce equation (18) to

\[
-C\frac{h}{\mu} + \mu \left(t_C \frac{\partial C}{\partial t C} + C\right) = 0,
\]

which implies that \( \mu > \frac{1}{h} \) because of \( \mu > 0 \) (see equation (20)) and \( t_C > 0 \). Therefore, (20) leads to \( N\frac{\partial U}{\partial G^S} > \frac{1}{h} \), which is compared to the first-best allocation (7) which can be written as \( N\frac{\partial U}{\partial G^F} = \frac{1}{h} \), using the property \( \frac{\partial U}{\partial V} = 1 \). Because the utility function is assumed to be strictly concave in \( G \) and \( \frac{\partial U}{\partial G^F} < \frac{\partial U}{\partial G^S} \), the proof of \( G^S < G^F \) is given.

This proof reconsidered a well known result of the public good provision in a distorted tax system, namely that the amount of the provided public good will decrease in a tax system with tax distortions compared to a first-best situation. This is explained because government spending becomes more costly if distortionary taxes are used to finance them which has been seen by comparing \( \mu \), the price for increasing the provision of the public good or in other words the shadow price of public revenue, with the marginal utility of of income \( \frac{1}{h} \) as seen in the individual’s first-order condition \( \frac{\partial U}{\partial V} = 1 \). It has been seen that this is higher in second-best \( (\mu < \frac{1}{h}) \) than in first-best \( (\mu = \frac{1}{h}) \) where lump-sum taxation is feasible. Therefore, additional revenue from the pollution tax in second-best is more valuable. If the environmental quality has the same characteristics as an ordinary public good, as well less environmental quality in second-best than in first-best would have to be expected.

Using a tax on the clean commodity for financing government spending \( (t^S_C > 0) \), it can be shown that the environmental quality is higher in the second-best allocation than in the first-best if the marginal revenue of a tax on the pollution
producing good is positive and vice versa, which can be written as $E^S > E^F \Leftrightarrow \frac{\partial R}{\partial t_D} > 0$.\(^8\)

The already shown properties $\mu > \frac{1}{h}$ and $\frac{\partial D}{\partial t_D} < 0$ hold. Equation (21) implies because of $\frac{\partial R}{\partial t_D} > 0$ and $\frac{\partial D}{\partial t_C} = 0$ that

$$\left[\frac{p^S_D - p^F_D}{h} - Ne' \left(\frac{\partial U}{\partial E^F} - \frac{\partial U}{\partial E^S}\right)\right] \frac{\partial D}{\partial t_D} < 0. \quad (23)$$

It is assumed that the environmental quality in second-best is lower than in first-best ($E^S \leq E^F$), which implies because of (2) that the consumed amount of the dirty commodity lies in the second-best allocation above the one in the first-best ($D^S \geq D^F$). Utility is strictly concave in $E$ and due to the assumptions a marginal change in the tax rate $t_D$ leads to the same decrease in the consumption of $D$ as a marginal change in the consumer price of $D$ ($\frac{\partial D}{\partial t_D} = \frac{\partial D}{\partial p_D} < 0$). It follows that $\frac{\partial U}{\partial E^F} - \frac{\partial U}{\partial E^S} < 0$ and $p^S_D - p^F_D < 0$.

Because of the properties $e' < 0$ and $\frac{\partial D}{\partial t_D} < 0$, the total term would be greater than zero which is a contradiction to equation (23). Therefore, $E^S > E^F$ is proven.

A distorted taxation system leads to the provision of less public good than in a tax system where lump-sum taxation is feasible, but at the same time higher environmental quality in second-best is offered. Because of the distortionary taxes, public revenues are more costly in the second-best allocation than in first-best. Therefore, provision of $G$ is more expensive and less of it will be provided. At the same time environmental quality improves in comparison to the first-best allocation because the by the pollution tax generated revenue becomes more valuable from a social point of view as seen above at the comparison of $\mu$.

What has to be emphasised at this point is that the obtained results depend on the condition that a marginal shift of the pollution tax increases the overall tax revenues ($\frac{\partial R}{\partial t_D} > 0$). This argument equals the proposition that the optimal pollution tax can be found on the normal side of the Laffer curve.

### 3.3.2 Model with Cross-Price Effects

The same proposition as before, namely that in a second-best taxation system less of the public good is provided ($G^S < G^F$) and the marginal revenue of the pollution taxation is positive ($\frac{\partial R}{\partial t_D} > 0$) will be extended by the assumption that cross-price effects $\frac{\partial C}{\partial t_D}$ and $\frac{\partial D}{\partial t_C}$ are significant. It will be shown that the

\[\text{The proof is done in the direction of } \leq.\]
environmental quality in second-best will still exceed the one in the first-best optimum \((E^S > E^F)\).

This more complex calculation can be bypassed by the assumption that the demand function \(D(p_C, p_D, p_L)\) is convex in prices \(p_C\) and \(p_D\). Because of the normalisation \(t_L = 0\), the price for labour \(p_L = h\) stays constant in first-best and second-best.

The assumed property of the demand function implies the condition

\[
(p_C^S - p_C^F) \frac{\partial D}{\partial t_C} + (p_D^S - p_D^F) \frac{\partial D}{\partial t_D} \geq D^S - D^F . \tag{24}
\]

Using this result in equation (21) leads to

\[
\frac{D^S - D^F}{h} - Ne' \left( \frac{\partial U}{\partial E^F} - \frac{\partial U}{\partial E^S} \right) \frac{\partial D}{\partial t_D} + \left( \mu - \frac{1}{h} \right) \frac{\partial R}{\partial t_D} \leq 0 . \tag{25}
\]

It is known from equations (20) and (7) that \(G^S < G^F \Leftrightarrow \mu > \frac{1}{h}\). Utility is strictly concave in \(E\) and \(\frac{\partial D}{\partial t_D} < 0\), as well as \(\frac{\partial R}{\partial t_D} > 0\) hold. The proof follows that of chapter 3.3.1.

It will be assumed again that \(E^S \leq E^F\). This implies as already shown \(D^S \geq D^F\). Therefore \(\frac{\partial U}{\partial E^S} - \frac{\partial U}{\partial E^F} < 0\). Because of the properties \(e' < 0\) and \(\frac{\partial D}{\partial t_D} < 0\) this result contradicts equation (25). Hence, it is proven that the first-best treatment leads to less environmental quality than the second-best \((E^S > E^F)\).

As seen, less provision of the public good \(G\) in second-best corresponds to more provision of the environmental quality \(E\) also in the case where cross-price effects are significant. This means that the discussion achieves the same results with and without the existence of cross-price effects.

The assumption \(\frac{\partial R}{\partial t_D} > 0\) still plays an important role. In the case of cross-price effects the impact on the overall revenue depends as well on the fact that the change of the tax rate influences the consumption of the other markets. Therefore, fulfilling this condition is more difficult.

### 3.3.3 Decomposition of the Effects

The occurring environmental effects of moving from the first-best to the second-best allocation can be determined by using the allocation \((C^Z, D^Z, V^Z, E^Z, G^Z)\), which is defined by the minimum of resources that are needed to reach the same utility level as in the second-best treatment if no distortionary taxes exist.
Therefore, the total environmental effect can be decomposed into

\[ E^F - E^S = (E^F - E^Z) + (E^Z - E^S), \]  

(26)

where the first parenthesis describes the income effect and the second the substitution effect.

The income effect is defined by the fact that first-best and second-best allocation lie on different indifference curves, which will not be observable in this model because quasilinear preferences are assumed. Therefore, an effect of an exogenous increase in income can not be determined and so, neutrality is assumed implicitly. Empirical research can give some clues about the income effects of different dirty commodities, but it has to be emphasised that if the income effect of pollution tends in one direction it will have a strong influence on the results.

The substitution effect originates from the occurring price change and has so due to the results a positive influence on the environmental quality. It can be decomposed again into a leisure substitution effect which reduces pollution, and a commodity substitution effect which increases pollution as will be extensively discussed in chapter 3.4.3.

### 3.3.4 Discussion of the Results

This chapter showed that the provision of the public good in a second-best setting is lower than in first-best and therefore the crowding-out hypothesis holds for it, but at the same time higher environmental quality in second-best treatment than in first-best is reached. Therefore, for the environment even if it has the same properties as a public good, namely it is non-rival and non-excludable, the hypothesis can not hold. Hence, it has to be emphasised that environmental quality and ordinary public good can not be treated in the same way in this model. Environmental quality does not compete with other public goods in a second-best world if pollution taxation is used. It has to be assumed that this result depends on the fact that the environmental policy generates revenues in the observed economy.

The results can be intuitively described with the argumentation of Gaube (2005, 2) that because of the distortionary tax of the second-best taxation system which replaces the lump-sum tax of the first-best, public funds are more costly. At the same time created revenues from the environmental protection are more valuable from a social point of view. Within this context a taxation system with distortionary taxes provides more room for environmental protection.
In the second-best treatment the idea to reduce the environmental quality from $E^S$ to $E^F$ would lead to a welfare gain and at the same time to a desirable amount of the environmental quality - at least the externalities would be internalized - can not hold. Due to the assumption of the positive marginal revenue of the pollution tax and the fact that revenue of the pollution tax in second-best is more valuable from a social point of view as seen above at the optimal provision of $G$, a reduction in environmental quality decreases public revenue more than the reachable welfare gain by reducing pollution taxation. Hence, the environment is optimally more protected in a second-best world.

It should be mentioned at this point that the results of this chapter are based on some strict assumptions and restrictions, but the model gives a clue that the use of distortionary taxes increases environmental quality.

3.4 Tax Distortions

In the last chapter the idea that distortionary taxes have a positive influence on the environmental quality has been given. This poses the question of how the optimal tax rates react within higher tax distortions and how these influence the environmental quality, which will be discussed in this chapter using the paper of Metcalf (2003) and its corresponding working paper (Metcalf, 2000).

As in the last chapter the normalisation $t_L = 0$ holds for this consideration. An additional specification of the environmental quality within the model will be used for the calculations. The public good $G$, will stand for all government services and contributes to the environmental quality, in other words, a part of the government spending also causes pollution. This fraction will be defined as $\gamma$ and therefore the environmental quality can be written as

$$E = e(ND + \gamma G), \quad e'(ND + \gamma G) < 0. \quad (27)$$

A homothetic subutility function for $C$ and $D$, namely $Q(C, D)$ will be used as in chapter 3.3, but here weak separability between the subutility and leisure instead of additive separability are assumed. $\sigma$, the log-linearised substitution elasticity in consumption of the commodities and therefore the preferences can be written as

$$\hat{C} - \hat{D} = \sigma(\hat{t}_D - \hat{t}_C), \quad (28)$$

where the hats stand for the proportional change of the commodities, namely $\hat{C} = \frac{dC}{C}$, $\hat{D} = \frac{dD}{D}$ and where $\hat{t}_D = \frac{dp_D}{1 + t_D}$ and $\hat{t}_C = \frac{dp_C}{1 + t_C}$ is the proportional change of the tax of the consumer price. Because of $p_C = 1 + t_C$ and $p_D = 1 + t_D$ it
follows that $\hat{t}_C = \hat{p}_C$ and $\hat{t}_D = \hat{p}_D$.

Constant government spending assumed, equation (14) and therefore the relationship between the optimal tax rates of $t_D$ and $t_C$ in second-best can be written (see appendix A.4) as

$$t_D = t_C + (1 - \epsilon t_C) \tau,$$

(29)

where $\epsilon$ is the uncompensated labour supply elasticity of this model.

As already seen in chapter 3.1 and 3.2, if there is no need for distortionary taxes, the optimal tax on the polluting commodity equals the Pigouvian level. If $t_C$ is required, the Pigouvian tax increment that is defined as the difference between the tax on the polluting good and the tax on the clean good, namely $(t_D - t_C)$ stays at the Pigouvian level, as long as labour is offered at a constant level, which means that the labour supply elasticity $\epsilon$ is zero. If $\epsilon t_C > 0$, then $t_D - t_C$ is smaller than $\tau$, which means that the Pigouvian tax increment falls short of the marginal social damage.

3.4.1 Increase in Government Services

The government services $G$ are treated as an exogenous variable in the following. It is assumed that the height of the government spending is set at a certain level and for achieving the necessary tax revenues the government chooses the tax rates in an optimal way, which means that they minimise the deadweight loss.

This assumption seems to be quite realistic because a political process leads to the decision of $G$. One could think that a newly elected government follows a different policy to the previous one and therefore a change in the public spendings will occur. However the reasons for higher government spending are described higher spending will lead to higher tax distortions in the treated model and its the impact on optimal tax rates as well on the environmental quality can be observed.

3.4.2 Impact on the Tax Rates

Equation (29) can be written as

$$t_D = \tau + (1 - \epsilon \tau) t_C,$$

(30)

where it is seen that $dG > 0$ implies that $sgn(dt_C) = sgn(dt_D) > 0$ as long as $1 - \epsilon \tau > 0$, which is in the following assumed to hold and Laffer curve effects are
still assumed to be absent for both tax rates.\textsuperscript{9} 

Due to the mentioned change of the government spending the total differential of equation (29) and therefore of the Pigouvian tax increment can be written as

\[ d(t_D - t_C) = -\epsilon \tau dt_C. \tag{31} \]

An increase of $G$ is accompanied by a fall of the Pigouvian tax increment, as long as $\epsilon > 0$. In other words, the difference between the used tax rates decrease if an increase in government spending is necessary.

It therefore results that increased public spending is not optimally financed by solely an increase of the pollution tax as mentioned in the double-dividend discussion.

### 3.4.3 Impact on the Environmental Quality

After the effects on the tax rates - that are accompanied by an increase in the government spending and therefore in tax distortions - are shown, the focus will be set again on the environmental quality.

If for example, the government spends the additional expenditures solely on clean commodities, this by itself would have a positive effect on the environmental quality and therefore the observed conclusions about the occurring effects would be influenced. To avoid this, it is assumed that the government spending will be transacted in the same proportion for clean and dirty commodities as done by the individuals and hence demand side effects can be ruled out. As already mentioned the fraction of the government spending for dirty commodities will be defined as $\gamma$ and therefore the amount of the dirty commodities due to government services can be written as

\[ \gamma G \equiv \frac{\pi_D}{\pi_C + \pi_D} G, \tag{32} \]

where $\pi_C = \frac{C}{hL}$ and $\pi_D = \frac{D}{hL}$ are the shares of the commodities of total production. The production shares $\pi_C$, $\pi_D$ and $\pi_G = \frac{G}{hL}$ sum up to one.

As seen in equation (27) environmental quality increases if $ND + \gamma G$ decreases. The total differential $dD + \gamma \frac{dG}{N} < 0$ can be rearranged to

\[ (1 - \pi_G) \hat{D} + \pi_G \hat{G} < 0. \tag{33} \]

The total differential of the production frontier (1), namely $dL = \frac{dC}{h} + \frac{dD}{h} + \frac{dG}{Nh}$

\textsuperscript{9}Using a high $\epsilon$ of 0.5, this expression is positive, as long as $\tau < 2$, which means that the marginal social damage of pollution does not exceed twice the production cost of the dirty commodity.
can be written as
\[ \hat{L} = \pi_C \hat{C} + \pi_D \hat{D} + \pi_G \hat{G} \]  
(34)
which says that a change in labour supply leads to changes in production. Therefore, equations (28) and (34) can be combined by substituting \( \hat{C} \) to
\[ (1 - \pi_G) \hat{D} + \pi_G \hat{G} = \pi_C \sigma (\hat{t}_C - \hat{t}_D) + \hat{L}. \]  
(35)

For statements about the change of the environmental quality due to an increase in government spending the right hand term, namely \( \pi_C \sigma (\hat{t}_C - \hat{t}_D) + \hat{L} \) has to be viewed. If this term is smaller than zero environmental quality improves as seen in equation (33).

To define the occurring effects, the term can be viewed as split. The first term can be declared as a commodity substitution effect, while the second term can be interpreted as a leisure substitution effect.

Since the Pigouvian tax increment falls, the commodity substitution effect, which says that the individuals will substitute from the clean commodity \( C \) to the dirty commodity \( D \), will be positive (see appendix A.4). One could explain this because the dirty commodity becomes relatively cheaper compared to the clean commodity. It has to be noted that the strength of the effect will depend on \( \sigma \), the elasticity of substitution. Obviously this commodity substitution effect will worsen the environmental quality of the economy because consumed clean commodities will be replaced by dirty ones.

Meanwhile the right hand term deals with the labour supply, namely
\[ \hat{L} = \epsilon \hat{w}, \]  
(36)
which will fall as long as the real wage \( w = \frac{h}{p_Q} \) falls and the labour supply elasticity \( \epsilon \) is not completely inelastic. \( h \) can be seen as the fixed gross wage because the labour market stays untaxed. \( p_Q \) is a price index of the consumption bundle \( Q(C, D) \).

The real wage will be reduced, if the tax rates of the commodity \( C \) as well as on \( D \) rise. Therefore, a change of the real wage can be written as \( dw = \frac{-\phi \hat{t}_C - (1 - \phi) \hat{t}_D}{L p_Q} \) and rearranged to
\[ \hat{w} = -\phi \hat{t}_C - (1 - \phi) \hat{t}_D, \]  
(37)
where \( \phi = \frac{p_C}{p_C + p_D} \), which is the fraction of the individuals’ spending on the clean commodity and therefore \( (1 - \phi) \) the spending on the dirty commodity.

Raised tax rates lead to a lower real wage and therefore the consumers sub-
stitute commodities by leisure, which does not contribute to pollution and is therefore assumed as clean in this model. Hence, the leisure substitution effect is working for the environment.

Consequently, the total effect on environmental quality depends on two effects within this model. On the one hand there is the commodity substitution effect, which has a negative influence on the environmental quality, and on the other hand the leisure substitution effect that decreases pollution. Which of these effects is stronger and will overlay the other depends obviously on the relative size of these two.

3.4.4 Comparative Static

Metcalf implemented parameter values for a more seizable discussion of the above mentioned effects. These values can be seen in table 1 (see appendix A.6.1).

As seen in table 2 (see appendix A.6.2) for an increased government revenue of 10% the Pigouvian tax increment falls for all values in the range of estimates. This means that the difference between the tax and therefore the price of the dirty and the clean commodity becomes smaller and the dirty commodity will become proportionally cheaper for all estimations.

For nearly all estimated values, the impact of the increase of the government revenues on the environmental quality can be emitted as positive within the range of estimation, except one case where a high elasticity of substitution and a low labour supply elasticity lead to an decline of environmental quality as can be found in table 3 (see appendix A.6.3).

3.4.5 Discussion of the Results

This chapter showed that a tax system with distortionary taxes has an influence on the tax rates as well as on the environmental quality. Higher distortions due to higher government spending lead on the one hand to a decrease of the difference between the used tax rates, and at the same time to less pollution and therefore a higher environmental quality. This confirms the already mentioned quote from Gaube (2005) that it seems that distortions in a taxation system offer more room for environmental protection. Within the discussed model the height of government spending could be seen as an indicator for environmental quality.

That the Pigouvian tax increment falls when higher government revenues are needed can be explained intuitively. Metcalf (2003, 318) notes that the optimal pollution tax consists out of a Ramsey tax component and an environmental component. Due to the assumed separability between leisure and consumption
goods it can be shown that the optimal Ramsey components of the consumption goods are equal. Higher public revenue increases the Ramsey tax component, while the weight on the environmental component falls, as seen in the decrease of the difference between the tax rates.

This result is not surprisingly because as known from taxation theory the excess burden of a tax is minimised by the Ramsey tax which becomes more important, if higher tax distortions exist. Hence, both commodity tax rates increase. If on the other hand the environmental tax component would be increased an additional excess burden would occur. Due to this consideration, the connection between the different optimal pollution tax rates in chapter 3.2 is evident.

Two different effects due to an increase of taxation play an important role for the discussion about the environmental quality within the model. There is, on the one side, a commodity substitution effect that increases the consumption of dirty commodities and a leisure substitution effect that lets the individuals substitute from consumption to leisure. As it has been seen, the labour market is as well affected through different channels and plays an important role for the discussion about the impact on the environment, even if it stays untaxed within this model. It can be concluded that solely the knowledge of a change in the optimal pollution tax rate will not be sufficient to conclude the impact on the environment.

Metcalf (2003, 320) emphasises that the assumption of leisure as clean is not accurate and extreme. This point is not totally understandable because consumption is defined as consuming clean and dirty commodities. If pollution producing commodities are consumed in the leisure time, these are already defined through consumption. Hence, leisure itself will not affect the environment.
4 Conclusion

This thesis discusses the connection between the optimal pollution taxation and its impacts on the environmental quality by mean of a general equilibrium model. That there can be found very few literature that deals with these effects - the obvious reason for using pollution taxes - is surprising.

The environmental quality in a second-best taxation system exceeds the first-best level. Additional tax distortions lead to a further reduction of pollution, while the environmental component of the optimal pollution tax decreases. This result can be seen as a contradiction to the mentioned double-dividend idea because the Ramsey component of a tax minimises the excess burden and is therefore crucial if a distorted tax system is viewed.

If the taxation system is not set optimally, higher tax distortions have to be expected. Due to the obtained results, it has to be assumed that the overall consumption will be reduced and has therefore a positive impact on the environmental quality. But, depending on the properties of the pollution producing commodities, possible impacts of an income effect could play an important role.

It should be noted that all obtained results concerning the optimal pollution tax and the environmental quality depend on the fact that an increase in the pollution tax rate leads to an increase in the overall public revenue. If this assumption falls the reached results can not hold. If as a starting point, the case where no tax on the pollution producing commodity exists would be considered, the assumption would hold easily. But because the discussion takes place at a pollution tax rate above the Pigouvian level, its fulfillment can be more difficult.

The result that the environmental quality will be higher if distortionary taxes exist than if lump-sum taxation would be feasible, while the provision of the public good will be reduced depends obviously on the fact that only pollution taxation is used as pollution abatement policy within the discussed model. If the opposite case is assumed, namely that the environmental policy causes cost and does not generate public revenue, no difference between the environment and a public good will be observable. Within the used models an additional pollution abatement instrument could be implemented as a fraction of the public good, but then the positive influence on the utility from the abatement also has to be taken into account. Even if mixed policy instruments as in reality have an impact on the obtained results it can be concluded that the use of pollution taxation leads to another treatment of the environment within policy making.

Within a taxation system, many different targets as competitiveness of the
economy or distributional concerns will have to be taken into account. How these affect the environment can not directly be answered within the discussed model, but it can be concluded that, if the deadweight loss of a taxation system is minimised a positive impact on the environment can be expected, but the occurring environmental effects should be still observed.
References


A Appendix

A.1 First-order conditions of (4) and (5)

Maximisation problem (4) implies \( \frac{\partial U}{\partial C} = \lambda (1 + t_C) \), \( \frac{\partial U}{\partial D} = \lambda (1 + t_D) \), and \( \frac{\partial U}{\partial V} = \lambda h (1 - t_L) \), while the first-order conditions of (5) imply \( \frac{\partial U}{\partial C} = \lambda \), \( \frac{\partial U}{\partial D} + \frac{\partial U}{\partial E} e' N = \lambda \), \( \frac{\partial U}{\partial V} = \lambda h \), and \( \frac{\partial U}{\partial G} = \lambda \frac{1}{N} \), where \( \lambda \) denotes the belonging Lagrange multiplier.

A.2 Derivation of (18), (19), and (20)

Using the utility function (17), the second-best maximization problem leads to

\[
\begin{align*}
t_C & : \frac{\partial W}{\partial t} + \frac{\partial W}{\partial E} \frac{\partial D}{\partial t} + \mu \frac{\partial R}{\partial t} = 0, \quad \text{(A.1)} \\
t_D & : \frac{\partial W}{\partial t} + \frac{\partial W}{\partial E} \frac{\partial D}{\partial t} + \mu \frac{\partial R}{\partial t} = 0, \quad \text{(A.2)} \\
G & : \frac{\partial W}{\partial G} - \mu \frac{1}{N} = 0, \quad \text{(A.3)}
\end{align*}
\]

where \( \mu \) is the Lagrange multiplier.

Roy’s identity leads to \( \frac{\partial W}{\partial C} = -C \lambda \) and \( \frac{\partial W}{\partial D} = -D \lambda \). The assumed utility function and \( t_L = 0 \) leads to \( \frac{\partial U}{\partial V} = 1 \). Therefore, \( \lambda = \frac{\partial U}{\partial V} h = \frac{1}{h} \). Inserting these results and the properties \( \frac{\partial W}{\partial E} = \frac{\partial U}{\partial E} \) and \( \frac{\partial W}{\partial G} = \frac{\partial U}{\partial G} \) into the first-order conditions leads to (18), (19), and (20).

A.3 Derivation of (21)

Add and subtract \( \frac{\partial R}{\partial D} \frac{1}{h} = (t_C \frac{\partial C}{\partial D} + t_D \frac{\partial D}{\partial D} + D) \frac{1}{h} \) to (19) leads to

\[
\begin{align*}
\frac{t_C}{h} \frac{\partial C}{\partial D} + \frac{t_D}{h} \frac{\partial D}{\partial D} + Ne' \frac{\partial U}{\partial E} \frac{\partial D}{\partial D} + (\mu - \frac{1}{h}) \frac{\partial R}{\partial D} = 0. \quad \text{(A.4)}
\end{align*}
\]

The first-best allocation can be implemented by the mean of its prices, namely \( p_L^F = h \), \( p_C^F = 1 \), and \( p_D^F = 1 + t_D^F = 1 - h Ne' \frac{\partial U}{\partial E} \). Hence, the second-best tax-rates are \( t_C^S := p_C^S - 1 = p_C^S - p_C^F \) and \( t_D^S := p_D^S - 1 = (p_D^S - p_D^F) + (p_D^F - 1) = (p_D^S - p_D^F) - h Ne' \frac{\partial U}{\partial E} \).

Using these as well as the property \( \frac{\partial C}{\partial D} = \frac{\partial D}{\partial C} \), which can be concluded from the symmetry of the Slutsky matrix because \( D(\cdot) \) and \( C(\cdot) \) are Hicksian demand functions, leads to equation (A.4).
A.4 The Commodity Substitution Effect

The differential of equation (30), namely \( dt_D = (1 - \epsilon \tau) dt_C \) can be written as

\[
\hat{t}_D = (1 - \epsilon \tau) \frac{1 + t_C \hat{C}}{1 + t_D \hat{C}} \equiv \Omega \hat{C},
\]

where \( \Omega < 1 \), if \( t_D > t_C \) and \( 1 - \epsilon \tau < 1 \). This implies \( \hat{t}_D - \hat{C} = (\Omega - 1) \hat{C} < 0 \). Therefore, \( \hat{C} - \hat{t}_D > 0 \).

A.5 Derivation of (29)

Assuming a constant level of government spending (\( dG = 0 \)), the change in consumption of \( C \) and \( D \) due to a change in \( t_D \) can be solved.

Using \( t_L = 0 \) and \( dG = 0 \), the total differential of the government budget constraint (9) can be written as \( dt_C C + t_C dC + dt_D D + t_D dD = 0 \) and rearranged to

\[
\frac{p_C C}{p_C C + p_D D} \left( \hat{C} + \frac{t_C}{1 + t_C} \hat{C} \right) + \frac{p_D D}{p_C C + p_D D} \left( \hat{D} + \frac{t_D}{1 + t_D} \hat{D} \right) = 0.
\]

(A.6)

Out of equations (34), (36), and (37) one gets

\[
\pi_C \hat{C} + \pi_D \hat{D} = -\frac{p_C C}{p_C C + p_D D} \hat{C} \epsilon - \left( 1 - \frac{p_C C}{p_C C + p_D D} \right) \hat{D} \epsilon.
\]

(A.7)

Using equations (A.6), (28) and (A.7) one has three equations depending on the variables \( \hat{C}, \hat{D}, \hat{t}_D \) and \( \hat{C} \), which can be solved as functions of \( \hat{t}_D \).

Substituting \( \hat{D}, \hat{C} \) equations (A.7) and (28) can be combined to

\[
\hat{C} = \frac{1}{C + D} \left( \hat{t}_D (\sigma D + \epsilon p_D D) + \hat{C} (-\sigma D - \epsilon p_C C) \right),
\]

(A.8)

\[
\hat{D} = \frac{1}{C + D} \left( -\hat{t}_D (\sigma C + \epsilon p_D D) - \hat{C} (\epsilon p_C C - \sigma C) \right).
\]

(A.9)

Inserting these results into (A.6) leads to

\[
\hat{C} = \frac{D}{C} \frac{p_D D (1 - \epsilon t_D) + C (-p_D (1 - \epsilon t_C) + \sigma (t_D - t_C))}{p_C C (1 - \epsilon t_C) + D (p_C (1 - \epsilon t_D) + \sigma (t_D - t_C))} \hat{t}_D.
\]

(A.10)

Implementing (A.10) into (A.8) and (A.9) leads to the general equilibrium response of a change in \( t_D \)

\[
\hat{C} = \frac{dC}{C} = \frac{D}{C} \frac{\sigma ((1 - \epsilon t_D) (p_C C + p_D D))}{p_C C (1 - \epsilon t_C) + D (p_C (1 - \epsilon t_D) + \sigma (t_D - t_C))}.
\]

(A.11)
and

\[ \dot{D} = \frac{dD}{D} = -\frac{\sigma(1 - \epsilon t_C)(p_C c + p_D d)}{p_C c (1 - \epsilon t_C) + D(p_C (1 - \epsilon t_D) + \sigma (t_D - t_C))}. \]  

(A.12)

Therefore, \( \frac{dC}{dD} = \frac{1-\epsilon t_D}{1-\epsilon t_C} \). Using this result in equation (14) leads to equation (29).

### A.6 Comparative Static

To show the general equilibrium effect of an increase in G on the variables \( C, D, L, w, t_c, \) and \( t_D \) equations (28), (34), (36), (37) and (A.5) are used, as well as the differentiated household budget constraint (A.6) where the change in \( G \) is taken into account, namely

\[ \frac{p_C c}{p_C c + p_D d} (\hat{t}_C + \frac{t_c}{1+t_c} \hat{C}) + \frac{p_D d}{p_C c + p_D d} (\hat{t}_D + \frac{t_D}{1+t_D} \hat{D}) = \frac{G}{NhL} \hat{G}. \]

### A.6.1 Table 1 (Metcalf, 2003, 320)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon )</td>
<td>0.30</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.30</td>
</tr>
<tr>
<td>( \pi_c )</td>
<td>0.30</td>
</tr>
<tr>
<td>( \pi_d )</td>
<td>0.40</td>
</tr>
<tr>
<td>( \pi_g )</td>
<td>0.30</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### A.6.2 Table 2 (Metcalf, 2003, 320)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \epsilon )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon = 0.15 )</td>
<td>-0.0030</td>
<td>-0.0030</td>
</tr>
<tr>
<td>( \epsilon = 0.30 )</td>
<td>-0.0066</td>
<td>-0.0066</td>
</tr>
<tr>
<td>( \epsilon = 0.45 )</td>
<td>-0.0110</td>
<td>-0.0109</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma = 0.5 )</td>
<td>-0.0030</td>
</tr>
<tr>
<td>( \sigma = 1.0 )</td>
<td>-0.0066</td>
</tr>
<tr>
<td>( \sigma = 2.0 )</td>
<td>-0.0065</td>
</tr>
</tbody>
</table>

This Table shows \( d(t_D - t_c) \) for a 10% increase in \( G \).
### A.6.3 Table 3 (Metcalf, 2003, 320)

Table 3

Impact of increased revenue requirement on dirty production

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.15</td>
<td>0.30</td>
<td>0.45</td>
</tr>
<tr>
<td>$\sigma$ 0.5</td>
<td>0.003</td>
<td>0.007</td>
<td>0.012</td>
</tr>
<tr>
<td>$\sigma$ 1.0</td>
<td>0.002</td>
<td>0.006</td>
<td>0.010</td>
</tr>
<tr>
<td>$\sigma$ 2.0</td>
<td>0.000</td>
<td>0.003</td>
<td>0.007</td>
</tr>
</tbody>
</table>

This table shows $d(ND + \gamma G)$ as a fraction of total output for a 10% increase in $G$. 
Abstract

This thesis deals with the connection between the use of pollution taxation and the impact on the environmental quality. Noting the popularity of pollution taxation within policy making, its effects on the environmental quality will be determined and evaluated. For this analysis a general equilibrium model that shows the influences of different markets is used.

Considering the difference between tax rates and the corresponding allocations, it is shown that the reached environmental quality is higher in a tax system where distortionary taxes are used than if lump-sum taxation would be available.

Additional public spending and therefore higher tax distortions lead to an increase in environmental quality, while the environmental component of the pollution tax decreases. Distorted taxation systems do not - different than from the double-dividend idea expected - favour pollution taxation as source for public revenue. Solely knowing the resulting change in the optimal pollution tax rate is not sufficient for determining the occurring impact on the environment.

It can be concluded that using pollution taxation in a tax system where the deadweight loss is minimised will lead to a desirable improvement of the environment.
Zusammenfassung

Die vorliegende Diplomarbeit beschäftigt sich mit dem Zusammenhang zwischen der Verwendung von Umweltsteuern und deren Auswirkungen auf die Umweltqualität. Da Umweltsteuern als Politikinstrument augenscheinlich sehr populär sind, werden die Effekte derselben bestimmt und bewertet. Für diese Analyse wird ein Gleichgewichts-Modell, das die Einflüsse verschiedener Märkte sichtbar macht, betrachtet.

Wird der Unterschied zwischen Steuerraten und den dazu gehörigen Allokationen berücksichtigt, kann gezeigt werden, dass die Umweltqualität in einem Steuersystem mit verzerrenden Steuern höher ist, als wenn die Verwendung von Pauschalsteuern möglich wäre.

Werden die Staatsausgaben und damit auch die auftretenden Verzerrungen in einem Steuersystem erhöht, ist eine Steigerung der Umweltqualität zu beobachten, obwohl die Umweltkomponente der optimalen Umweltsteuer abnimmt. Daher wird diese, anders als von der Diskussion über eine doppelte Dividende der Umweltsteuer erwartet, in einem verzerrten Steuersystem nicht für die Generierung von Staatseinnahmen bevorzugt. Aus der resultierenden Änderung der optimalen Umweltsteuerrate kann nicht auf die auftretenden Umwelteffekte geschlossen werden.

Aus der Modellbetrachtung kann gefolgt werden, dass die Verwendung von Umweltsteuern in einem Steuersystem, welches den Wohlfahrtsverlust minimiert, einen positiven Einfluss auf die Umweltqualität hat.
Curriculum Vitae

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10/2008 - 06/2009 Study of Economics, University of Alicante, Spain
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Publications