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Price Signaling Quality In Vertical Industry Structures

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Abstract

In most industries we observe vertical structures where retailers sell the manufacturers' products to consumers. The aim of this paper is to create a model which shows that separation has an advantage that cannot be imitated by vertical integration. Thus, we allow the manufacturer to choose to sell either to a retailer or directly to consumers. When a monopoly manufacturer is the only agent that knows the quality of its product and sells it directly to consumers, only a separating equilibrium where the price signals the quality satisfies the Intuitive Criterion (IC). We show that vertical separation as a way to credibly hide the knowledge of quality can enable a pooling equilibrium that satisfies the IC and generates higher profits.

1 Introduction

In most industries we observe vertical structures where manufacturers vertically separate. Here manufacturers don't sell their goods directly to consumers but to a retailer who sells the product to consumers. Vertical separation is generally seen as costly and inefficient because manufacturer *and*

*The initial idea of this paper is based on notes from Maarten Janssen and Santanu Roy. The author's contribution to this paper is the formulation of the introduction, reviewing the existing literature, proofing that firms can have an incentive to vertically separate, running numerical simulations, comparing across equilibria, and illustrating the findings.

retailer optimize their profits. Hence, it creates two margins – the *double marginalization problem* arises. Recent empirical studies showed indeed mixed effects of vertical separation but rather increasing prices (Mizutani and Uranishi 2013; Wilson 2015).

To explain vertical separation (and integration), incentive aspects have been initially considered in theory (see Mathewson and Winter 1984; Rey and Tirole 1986). More recent theoretical works emphasized strategic motives why firms engage in vertical separation. Weaker competition in the form of tacit collusion is shown to arise from vertical separation (see Bonanno and Vickers 1988; Pagnozzi and Piccolo 2012). In this view vertical separation is a strategic decision of a manufacturer to reduce competition. Pagnozzi and Piccolo (2012) studied vertical separation with private contracts. They found that selling through a retailer decreases competition and, consequently, increases the manufacturers' profit.

The purpose of the present paper is to study another motive why a manufacturer wants to vertically separate: Selling through a retailer as a way to credibly hide the knowledge of quality to avoid inefficiencies generated through price signaling. High prices as a signal for high quality can occur in markets where manufacturers are informed about the quality of their good but consumers are not. From Bagwell and Riordan (1991) we have already learned that in monopolisitic markets with such asymmetric information of quality a separating equilibrium exists. In such a separating equilibrum the high quality product is sold for a high price and the low quality product for a low price. Thus, the price reveals in this equilibrium information on the product's quality. Moreover, Bagwell and Riordan (ibid.) have shown that only a separating equilibrium satisfies the *Intuitive Criterion* (IC) by Cho and Kreps (1987). The IC is a equilibrium refinement technique that requires beliefs to be 'reasonable'. To signal high quality through a price, the high quality manufacturer distorts the price so high such that the low quality manufacturer doesn't want to mimic this price. However, such a price distortion generates inefficiencies.

In this paper we consider if selling indirectly through a retailer who is uniformed about the product's quality, is a way to overcome the inefficient outcome of a separating equilibrium. Our idea is that vertical separation allows the existence of a *pooling equilibrium* satisfying the IC. A pooling equilibrium can't exist when the manufacturer sells its product directly to the consumer. In such a pooling equilibrium, both types of quality manufacturer sell their product to an unique wholesale price to a retailer. The benefits from a pooling equilibrium outcome can overweight under certain conditions the costs of selling indirectly through a retailer. Then for such cases, we show an additional motive why a manufacturer decides to vertically separate. Studying this, we retain the standard price signaling framework by Bagwell and Riordan (1991) and extend it with a retailer as intermediary between manufacturer and consumers.

This paper is organized as follows: The next section 2 gives an overview on the related literature on signaling games focusing on price signaling. Section 3 outlines the basic model with and without vertical separation. Following Bagwell and Riordan (ibid.), we analyze in section 4 the case where the manufacturer has decided to vertically integrate and sells its product directly to consumers. In section 5 we add a monopolistic retailer to our model to show that a pooling equilibrium under vertical separation can satisfy the IC and generate larger profits than under vertical integration. Using these theoretical insights, we numerically simulate our model in section 6. Finally we conclude in section 7.

2 Literature Review

With his job market game, Spence (1973) was the first who provided a game theoretic analysis of a signaling game. In his model the ability of a job applicant is a private information that is not observable for the recruiter. Only education can be observed by all players but it has no improving effects on the applicant's ability. As education is assumed to be more costly for applicants with lower ability, Spence (ibid.) concludes that signaling ability through education allows applicants with higher ability to separate from them with lower ability.

Since Bagwell and Riordan (1991) many scholars have studied price signaling adopting their theoretical framework where quality is given endogenously. This is also the basis for the model of the present paper. For Bagwell and Riordan (1991) quality is simply either high or low. To signal quality, the high quality type distorts the price so high such that the low type doesn't want to mimic. To make this possible the costs for high quality have to be larger than the costs for low quality. As they allow for fractions of informed consumers, they find that the high price declines when repeating this game. Daughety and Reinganum (2008) defined quality as a continuous space between lowest and highest quality. In their model a monopoly manufacturer either chooses to signal quality through pricing or to disclose quality. They find that if disclosure costs are sufficiently high, the manufacturer decides to signal the product's quality through the price. In our model we don't allow the manufacturer to disclosure the product's quality, so we implicitly assume that the costs for disclosure are high.

Signaling private information on quality through prices appears not only in monopolies, also in competitive market structures with incomplete information manufacturers have an incentive to signal their quality through prices (Janssen and Roy 2010; Daher, Mirman, and Santugini 2012). Extending Bagwell and Riordan's (1991) model to more than one manufacturer, Janssen and Roy (2010) find that even in high competitive markets manufacturers have incentives to signal quality through pricing. Another insight they show is that price signaling can generate more market power for both types, high as well as low quality. Daher, Mirman, and Santugini (2012) studied price signaling in a classical Cournot model. Interesting in their theoretical approach is that, as manufacturers set here quantities and not prices, only one market price exists. So manufacturers can only incompletely control their price signal because the market price depends on the quantity of all manufacturers. Although, we study no competition and allow the monopoly firm only to charge a price, selling indirectly through a retailer can be also considered as an incompelete control over the manufacturer's price signal. Daher, Mirman, and Santugini (ibid.) find an unique equilibrium where under certain conditions the profits equal profits of an environment without incomplete information where firms collude. Hence, in a Cournot competition price signaling can decrease competition and increase manufacturers' profits.

Bontems and Mahenc (2014) studied a game where a monopoly manufacturer sells its product to a monopoly retailer through a two-part tariff. Here both, the manufacturer and the retailer, know the quality of the product. Without employing any equilibrium refinement, such as the IC, Bontems and Mahenc (ibid.) find that a vertical contract leads to a unique final price. We follow their idea of vertical separation as a way to avoid an inefficient separating equilibrium where the manufacturer distorts the price but use a different theoretical approach. First, we assume incompleteness and imperfection. In our model only the manufacturer is informed about the quality. Further, the wholesale price charged by the manufacturer to the retailer is only observable by the retailer. Considering real life, these assumptions seem to be plausible. Second, in our model the manufacturer charges a wholesale price and the retailer a retailprice. Third, unlike Bontems and Mahenc (ibid.) we apply the IC for equilibrium selection.

Considering imperfection, Bagwell (1995) has shown, that in pure strategy games where other players (at least slightly) imperfectly observe the first-mover's choice, the advantage of committing actions or moving-first is eliminated. The advantage of moving first is for instance in a Stackelberg duopoly already eliminated when adding the slightest degree of imperfection in the observability. Instead of the Stackelberg outcome, adding imperfection the Cournot outcome emerges as a sequential equilibrium outcome (ibid.). Indeed, in our model the imperfect information through vertical separation with which consumers are confronted, can result under certain parameter settings in a pooling equilibrium. This pooling equilibrium may generate higher profits to the first-mover, that is in our case the manufacturer. So imperfection is in our model not a disadvantage but applied to emerge strategic advantages.

Haan, Offerman, and Sloof (2011) studied a signaling model with a similar setup as the job market game of Spence (1973). The signal is here not send by a price but through a costly message. In extention to Spence's standard signaling model, Haan, Offerman, and Sloof (2011) added noise that systemically increases signaling costs of the high quality type. Theoretically they showed that adding noise reduces the number of equilibria. For a low level of noise, separating equilibria may completely disappear. A pooling equilibrium in which both types don't send any signal indeed always exists (Haan, Offerman, and Sloof 2011). In line with their theoretical findings, Haan, Offerman, and Sloof (ibid.) showed in an experiment that subjects tend more to a pooling outcome when the level of noise increases. Vertical separation can be also considered as a cost increasing noise. In our model the manufacturer can't send its signal directly to consumers under vertical separation. Moreover, the retailer reacts with higher prices to deviations from the manufacturer. Then signaling quality through prices turns out to be more costly than under vertical integration.

3 Basic Model

The timing of our model is the following: At the beginning a monopoly manufacturer (M) decides either to vertically integrate or vertically separate. In the first case M sells its products directly to consumers. If M decided to vertically separate, then the product is sold indirectly to consumers through a retailer. It is important to note that at this stage the quality of the product is unknown to M.

If M has decided to vertically integrate, we first model a game according to Bagwell and Riordan (1991) where M sells directly to the consumers. The quality of the good is heterogeneously given. Quality is with probability α high and with probability $1 - \alpha$ low. We assume α to be in $(0, 1)$. All consumers value the quality similar. M faces marginal costs of c_H if quality is high and c_L if quality is low. We assume producing high quality to be more costly than low quality which we set for simplicity equal to zero.

$$c_H > 0 \text{ and } c_L = 0$$

M observes its quality type and charges a price p. Consumers obsverve this price p and adopt their beliefs about the quality based on p. We denote beliefs that high quality is charging p as $\mathbb{P}(high|p) = \mu$. If μ is zero (one) consumers belief that p is comming from low (high) quality type. Bagwell and Riordan (1991) allow in their model for a fraction of ex-ante informed consumers. In our basic model all consumers have no ex-ante information about the quality.

We assume demand to be (linear) downward sloping in p. If quality is fully revealed – either through price signaling or full information – consumers demand

$$D^H(p) = H - p > D^L(p) = L - p > 0 \quad \forall p$$

units for high (H) respectively low (L) quality. Otherwise, without having any information on the quality, consumers demand

$$D^*(p) = \alpha D^H(p) + (1-\alpha)D^L(p) = \alpha H + (1-\alpha)L - p$$

After solving the game where M has decided to vertically integrate in section 4, we consider in section 5 the case where M has decided to vertically separate. Keeping all other assumptions unchanged, we add here a monopoly retailer (R) who buys for a wholesale price w the product from M and sells it to consumers. We denote the retail price R charges to consumers as $p(w)$. For simplicity, we assume that R has no additional costs and also selling through R brings no benefits to the consumers.

The timing is in the case where M decided to vertically separate the following: After observing the quality, M charges a wholesale price w. Then R observes w and charges the retail price $p(w)$ to consumers. Finally, consumers observe $p(w)$ on which they adopt their beliefs about the quality and demand as before in the case of vertical integration either $D^H(p(w)), D^L(p(w))$ or $D^*(p(w))$. It is important to note, that again only M is informed about the quality. Neither R nor the consumers have any information about it. Moreover, we add incomplete information by assuming that the wholesale price w charged by M is only observed by R and not by consumers.

Considering manufacturers that introduce from time to time new products and that have long-term contracts with their retailers, makes the timing assumption of our game plausible. Adding incomplete information is as well realistic as in general consumers are not informed about the wholesale price.

In line with Bagwell and Riordan (1991), we make in both games use of the IC by Cho and Kreps (1987) for equilibrium selection. This technique rules-out equilibria by requiring beliefs to be 'reasonable'. The IC is usually applied to models where one player sends a signal and one player receives it. In the case M decided to vertically integrate, we can make use of the IC in this standard way. Under vertical separation where we additionally have R who is a receiver as well as a sender of a signal, however, using the IC turns out to be more complex. How the IC is modified for the case of vertical separation, we explain and analyze in subsection 5.3. Comparing M's profits these two subgames, allows us finally to show in subsection 5.4 that M can have an incentive to vertically separate.

4 Vertical Integration

In this section we consider the case where M has decided not to vertically separate. M sells its product here directly to consumers. Analyzing this case, we first investigate the possibility for a pooling equilibrium. Thus, suppose there exists a pooling equilibrium where all quality types charge the same price $p_H = p_L = p^*$ and make following profits:

$$\begin{aligned}\Pi_L^*(p^*) &= (p^* - c_L)D^*(p^*)\\ \Pi_H^*(p^*) &= (p^* - c_H)D^*(p^*)\end{aligned}$$

where $D^*(p) = \alpha D^H(p) + (1-\alpha)D^L(p) = \alpha H + (1-\alpha)L - p$. For a pooling equilibrium $\alpha H + (1-\alpha)L > p^* \geq c_H$ should clearly be satisfied. By $c_H > c_L = 0$ we have $\Pi_L^*(p^*) > \Pi_H^*(p^*) \geq 0$. Since $p^* D^H(p^*) > p^* D^*(p^*) = \Pi_L^*(p^*)$ and the existence of a price $\hat{p}$ that generates a profit of $\hat{p}D^H(\hat{p}) < \Pi_L^*(p^*)$, e.g. $\hat{p} \geq H$ where $\hat{p}D^H(\hat{p}) \leq 0$, and by the continuity of demand, there exits a price $\tilde{p} > p^*$ such that:

$$\Pi_L(\tilde{p}, \mu = 1) = \tilde{p} D^H(\tilde{p}) = \Pi_L^*(p^*)$$
$$\Pi_H(\tilde{p}, \mu = 1) = \Pi_H^*(p^*) + c_H \left[D^*(p^*) - D^H(\tilde{p}) \right] > \Pi_H^*(p^*)$$
$$\text{or } D^*(p^*) > D_H(\tilde{p})$$

then according to the IC after observing such a price $\tilde{p}$ consumers' beliefs should be $\mathbb{P}(high|\tilde{p}) = 1$. In that case high quality type wants to deviate and obviously no pooling equilibrium satisfies the IC.

4.1 Separation Equilibrium

Let us now consider a separating equilibrium when M decided to sell its product directly to consumers. Here high and low quality types charge different prices, i.e. $p_H^* \neq p_L^*$. As low quality type then only serves to the low demand $D^L(p) = L - p$ it maximizes profits according to that and charges the monopoly price under full information $p_L^* = p_{L,L}^M = \frac{L}{2}$. Further, as $c_H > c_L = 0$ and so high type's profit $\Pi_H(p)$ strictly increases in p when $p \leq p_L^*$, in any equilibrium it must hold that $p_H^* \geq p_L^*$. Hence, in any *separating equilibrium* prices are $p_H^* > p_L^*$.

To have a separating equilibrium low quality type shouldn't have any incentive to mimic high quality type. So its profit has to be smaller when mimicking

$$\Pi_L(p_H^*, \mu = 1) \leq \Pi_L(p_L^*, \mu = 0) \tag{1}$$

Setting (1) equal we can derive

$$\overline{p}_L = \frac{H + \sqrt{H^2 - L^2}}{2}$$
$$\underline{p}_L = \frac{H - \sqrt{H^2 - L^2}}{2}$$

Obviously for any price between the upper and lower root, $p \in (\underline{p}_L, \overline{p}_L)$, low quality firm makes higher profits when mimicking. Consequently in a separating equilibrium high quality type has to charge a price $p_H^* \geq \overline{p}_L$ or $p_H^* \leq \underline{p}_L$. Under full information high quality firm charges its monopoly price $p_{H,H}^M = \frac{H+c_H}{2}$. As $p_H^* > p_L^*$ and $p_L^* > \underline{p}_L$[1] we can according to Bagwell and Riordan (1991, p. 228) establish the following necessary condition

Proposition 1. *$p_H^* = max\{\overline{p}_L, p_{H,H}^M\}$ and $p_L^* = p_L^M$ are the only separating equilibrium prices satisfying the IC*

Proof. We know already that low quality sets $p_L^* = p_{L,L}^M$, so suppose $p_H^* \neq max\{\overline{p}_L, p_{H,H}^M\}$. Then if high quality firm charges a price $p_H^* \in (\underline{p}_L, \overline{p}_L)$, low quality firm wants to mimic its counterpart and the IC fails. Also in the case when $p_H^* = p_{H,H}^M < \overline{p}_L$, low quality firm wants to mimic. If $p_H^* = \overline{p}_L < p_{H,H}^M$, high quality firm wants to deviate and charge $p_H^* = p_{H,H}^M$ which generates the highest profits. By the assumption $c_H > 0$ we further see that $\Pi_H(\overline{p}_L, \mu = 1) = \frac{L^2}{4} - \frac{H-\sqrt{H^2-L^2}}{2}c_H > \frac{L^2}{4} - \frac{H+\sqrt{H^2-L^2}}{2}c_H = \Pi_H(\underline{p}_L, \mu = 1)$ □

Now we have to analyze under which conditions such a separating equilibrium that satisfies the IC exists. Clearly, high quality firm has an incentive to charge $p_H^* = max\{\overline{p}_L, p_{H,H}^M\}$ iff

$$\Pi_H(p_H^*, \mu = 1) > \Pi_H(p_{L,H}^M, \mu = 0) \tag{2}$$

where charging the price $p_H = \frac{L+c_H}{2}$ in the left side of this expression generates the highest profits when high type is believed to be low quality. Together with setting (1) equal we can derive following condition for the existence of a separating equilibrium that satisfies the IC

$$\begin{aligned} D^L(p_{L,L}^M) - \frac{c_H}{4} &\geq D^H(p_H^*) \\ \text{or } p_H^* &\geq H - \frac{2L - c_H}{4} \end{aligned} \tag{3}$$

Now by our assumption $H > L > 0$ and as it should be that $p_H^* \leq H$, a separating equilibrium that satisfies the IC exists iff $c_H \leq 2L$. As high quality type would not want to charge a price below its monopoly price $p_{H,H}^M$,

[1]Consider $p_L^* \leq \underline{p}_L$ then $\frac{L}{2} \leq \frac{H-\sqrt{H^2-L^2}}{2} \Leftrightarrow H \leq L$ which contradicts our assumption $H > L$. So $p_H^* > p_L^*$ and $p_L^* > \underline{p}_L$

we clearly have then $p_H^* \in [p_{H,H}^M, H]$. In the case that $p_{H,H}^M$ is larger than $\overline{p}_L$, which can be the case either if c_H is large or $H - L$ is small enough, high quality type sets price to $p_{H,H}^M$. Otherwise, when $p_{H,H}^M$ is smaller than $\overline{p}_L$, high quality type charges in a separating equilibrium $\overline{p}_L$ because then profits decrease (when believed to be of high quality) in p. Hence, high quality type's equilibrium price is

$$p_H^* = \begin{cases} p_{H,H}^M & \text{if } c_H \geq \sqrt{H^2 - L^2} \\ \overline{p}_L & \text{else} \end{cases}$$

Separating equilibrium profits when both types of quality charge different prices can now be expressed as

$$\begin{aligned} \Pi_L^*(p_L^*, \mu = 0) =& \frac{L^2}{4} \\ \Pi_H^*(p_H^*, \mu = 1) =& (p_H^* - c_H)(H - p_H^*) \end{aligned}$$

M's expected profit in the case of vertical integration is then $\alpha\Pi_H^* + (1 - \alpha)\Pi_L^*$. M decides to vertically separate if it generates higher profits than under vertical separation. In subsection 5.4 we show that M can generate higher profits under vertical separation than under vertical integration.

Summarizing this section, in line with Bagwell and Riordan (1991) we find that on the one hand when M sells its product directly to consumers, no pooling equilibrium satisfying the IC exists. That means that both types of quality will never want to charge a unique price in the case of vertical integration. On the other hand, we have shown that for this case a separating equilibrium exits. Moreover, under certain conditions, we have derived, a separating equilibrium that satisfies the IC. Hence, when M decided to vertically integrate, M signals quality through prices. Then an inefficient outcome appears where high quality type distorts the price that high such that low quality type doesn't want to mimic.

5 Vertical Separation

Now we study the case where M doesn't sell its product directly to consumers. It vertically separates by selling its product to a monopoly retailer (R). We assume that both R and consumers don't know the quality of the product. Under vertical separation M first charges a wholesale price w to R. Then R takes w as given and sets the retail price $p(w)$. R has no additional costs. Consumers can only observe R's retail price $p(w)$ and build their beliefs based on that. Vertical separation provides no benefits for consumers.

Regarding the situation when M decided to sell its product via R instead of selling it directly to consumers, we show in this section, that under certain conditions a pooling equilibrium can exist. Additionally, we show in the subsequent subsections that a pooling equilibrium in the case of vertical separation can satisfy the IC and generate higher profits than a separating equilibrium when M sells directly to consumers. A separating equilibrium is for the case of vertical separation irrelevant, as it generates strictly smaller profits through double marginalization than in the case of vertical integration. So, if vertical separation results in a separating equilibrium, M will never decide to sell its product through R.

In a pooling equilibrium under vertical separation M charges w^* regardless of quality type and R sets a retail pricing strategy $p^*(w)$ such that no one has an incentive to deviate. In such an equilibrium M makes a profit of $\Pi^*_{M,i} = (w^* - c_i)D^*(p^*(w^*))$, $i = \{Low, High\}$, and R earns $\Pi^*_R(w^*) = (p^*(w^*) - w^*)D^*(p^*(w^*))$.

Consumers update their beliefs after observing a retail price $p(w)$. We assume now that consumers belief the product to be low quality if they observe a retail price $p(w) \neq p^*(w^*)$, i.e. $\mathbb{P}(High|p) = 0 \ \forall p \neq p^*(w^*)$. Then after observing $p(w)$ consumers demand

$$D(p(w)) \quad = \quad \begin{cases} D^*(p(w)) = \alpha H + (1-\alpha)L - p(w) & \text{if} \quad p(w) = p^*(w^*) \\ D^L(p(w)) = L - p(w) & \text{else} \end{cases}$$

Given these beliefs, R makes $\Pi^*_R(w) = (p(w) - w)D^*(p(w))$ when charging $p(w) = p^*(w^*)$. For any retail price $p(w) \neq p^*(w^*)$ consumers demand $D^L(p(w))$. Then the best price R can charge other than $p^*(w^*)$, is the monopoly retail price for low demand $p_L^M(w)$, which we define as $p_L^M(w) = \frac{L+w}{2}$. As demand for low quality is $L - p(w)$, clearly R will never want to charge retail price $p_L^M(w) > L$. So we have that $p_L^M(w) \in [\frac{L}{2}, L]$. Thus, R can for any w guarantee itself a profit of $\Pi_R^M(p_L^M(w)) = (p_L^M(w)-w)(L-p_L^M(w)) = \frac{(L-w)^2}{4}$ $\forall w \leq L$. Consequently, to establish a pooling equilibrium the following condition on R's profits has to be satisfied

$$\Pi_R^*(p^*(w^*)) \geq \Pi_R^M(p_L^M(w)) \tag{4}$$

In equilibrium, where $w = w^*$, expression (4) must hold in equality, as M would otherwise want to deviate and set $w > w^*$ while R wouldn't want to deviate from $p^*(w^*)$ in response to M's deviation. Setting $w = w^*$ and (4) equal we can derive

$$p_{1,2}^*(w^*) = \frac{\alpha H + (1-\alpha)L + w^* \pm \sqrt{(\alpha H + (1-\alpha)L - w^*)^2 - (L-w^*)^2}}{2}$$

The pooling demand is obviously higher for the smallest of these two values. The smallest value, i.e. $min\{p_1^*(w^*), p_2^*(w^*)\}$, is in equilibrium smaller or equal than $p_L^M(w^*)$ for all $w^* \leq L$. As the smallest value allows us to sustain easier a pooling equilibrium, we define the equilibrium retail price as

$$p^*(w^*) \equiv min\{p_1^*(w^*), p_2^*(w^*)\} = p_1^*(w^*) \quad \forall w^* \leq L$$

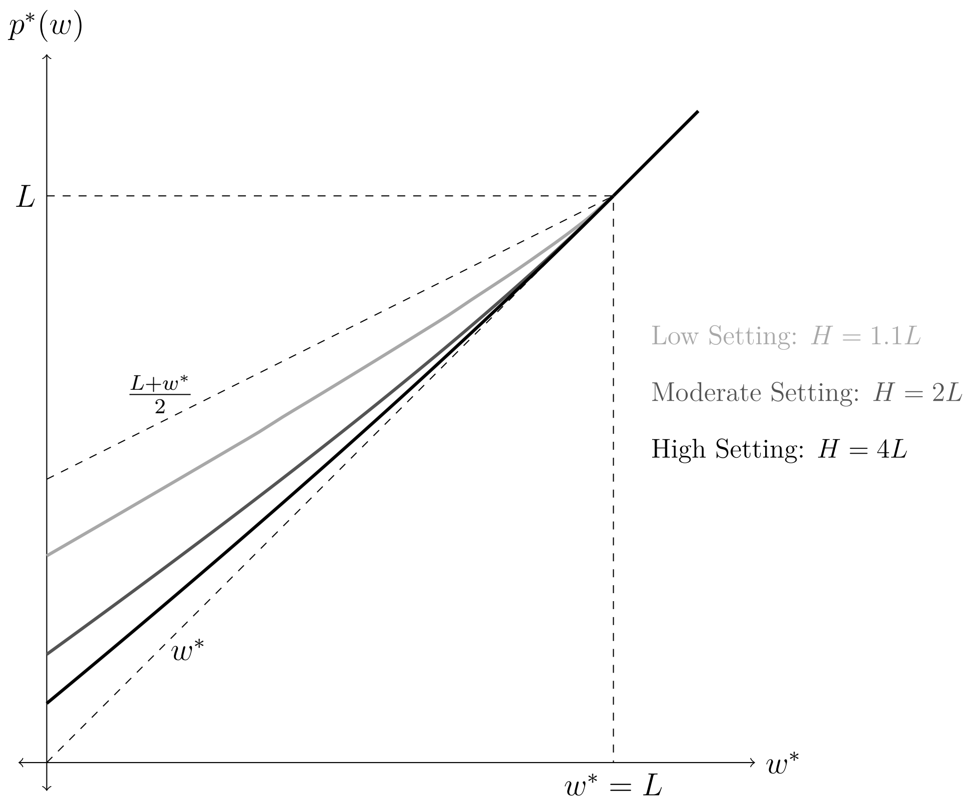
If $w^* > L$ then $p^*(w^*)$ has obviously to be equal to w^* such that expression (4) holds in equality. Otherwise R would generate $\Pi_R^M(p_L^M(w)) = 0$ but $\Pi_R^*(p^*(w^*)) > 0$. So we can summarize the equilibrium retail price $p^*(w^*)$ as following function of the equilibrium wholesale price w^*

$$p^*(w^*) = \begin{cases} p_1^*(w^*) & \text{if} \quad w^* \leq L \\ w^* & \text{if} \quad w^* > L \end{cases}$$

Figure 1 illustrates this function $p^*(w^*)$ for $\alpha = 0.5$ and three levels of H.

It shows that if H is close to L then $p^*(w^*)$ equals $p_L^M(w)$. In the other case, when H becomes large, then it converges to w^*.[2] Disregarding the level of H, the equilibrium retail price equals the equilibrium wholesale price w^* for all $w^* > L$.

Figure 1: Retailer Equilibrium Price as a Function of w^* ($\alpha = 0.5$)



5.1 Retailer's Equilibrium Strategy

In the upcoming subsection we shed light on R's equilibrium strategy $p^*(w)$. First, we analyze $p^*(w)$ for a $w < w^*$. In the left neighborhood of w^* (where $p^*(w^*) < p_L^M(w)$) R doesn't want to deviate from $p^*(w^*)$ because still charging the equilibrium retail price gives a profit of $(p^*(w^*) - w)D^*(p^*(w^*))$ which

[2] Similarly $p^*(w^*)$ shifts to $p_L^M(w)$ if α is close to 0 and to w^* if α is close to 1.

is larger than the best possible alternative, that is $p_L^M(w)$. When charging $p_L^M(w)$ R makes a profit of $(p_L^M(w) - w)(L - p_L^M(w)) = \frac{(L-w)^2}{4}$. Although in equilibrium both profits are equal, it can be shown that at $w = w^*$ the derivative of the equilibrium profit equals $-D^*(p^*(w^*))$ which is strictly smaller than $-D^L(p_L^M(w^*))$, the derivative of the deviation profit, as $D^*(p) > D^L(p)$ and $p^*(w^*) \leq p_L^M(w^*)$. Hence, R's response to a w in the left neighborhood of w^* is $p^*(w^*)$.

Now we consider the case where R observes a wholesale price $w < \tilde{w} < w^*$. Therefore we denote $\tilde{w}$ as a price where $\Pi_R^*(p^*(w^*)) = \Pi_R^M(p_L^M(w))$. Obviously, after observing such a wholesale price $w < \tilde{w}$, R wants to deviate and choose $p_L^M(w)$. By setting equation (4) equal we can derive $\tilde{w}$ as the smaller root of w of this expression as

$$\tilde{w} = L - 2D^*(p^*(w^*)) - 2\sqrt{\alpha D^*(p^*(w^*))(H - L)} \tag{5}$$

We note, that when such a wholesale price $w < \tilde{w}$ exists, R's equilibrium strategy is discontinuous at $w = \tilde{w}$. As charging $p_L^M(w)$ at $w < \tilde{w}$ can only be profitable if the demand at this wholesale price $D_L(p_L^M(w))$ is sufficiently larger than $D^*(p^*(w^*))$ – to compensate the loss of margins – and as $D^*(p) > D^L(p)$, we have to have a jump at $w = \tilde{w}$. Hence, R's equilibrium strategy is not continuous at $w = \tilde{w}$.

Second, for a wholesale price $w \leq L$ in the right neighborhood of w^* R would want to deviate and charge $p_L^M(w))$. Using the same argument as for w in the left neighborhood of w^*, it can be easily shown that here deviating and choosing $p_L^M(w))$ yields to a higher profit than when sticking to $p^*(w^*)$. At $w = w^*$ both profits, equilibrium and deviation, are equal, but the derivative of the equilibrium profit equals $-D^*(p^*(w^*))$ which is strictly smaller than $-D^L(p_L^M(w^*))$ as $D^*(p) > D^L(p)$ and $p^*(w^*) \leq p_L^M(w^*)$. Hence, R's response to a w in the right neighborhood of w^* is $p_L^M(w)$ as long as it is (sufficiently) smaller than L. Setting equation (4) equal we can derive now

$$w^* = L - 2D^*(p^*(w^*)) + 2\sqrt{\alpha D^*(p^*(w^*))(H - L)} \tag{6}$$

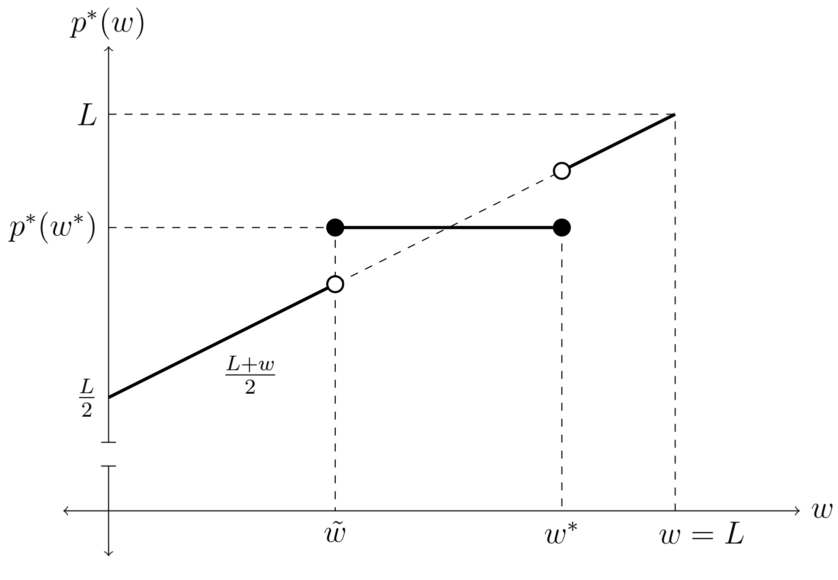
From this expression we see again that if $p^*(w^*) = L$ then we have $w^* = p^*(w^*)$.[3] This holds also for all $\alpha H + (1-\alpha)L > p^*(w^*) > L$, as then in equilibrium $w^* = p^*(w^*)$ has to be satisfied as otherwise expression (4) wouldn't hold in equality. Then at $w = w^* = L$ we have $p_L^M(w)) = L$ which equals $p^*(w^*)$ and gives zero profits as the margins are zero, $p^*(w^*) - w^* = 0$. Hence, for $p^*(w^*) \geq L$ R chooses $p^*(w^*)$.

Summarizing, we can characterize R's equilibrium strategy as follows

$$p^*(w) \quad = \quad \begin{cases} p^*(w^*) & \text{if} \quad w \in [\tilde{w}, w^*] \vee w \geq L \\ p_L^M(w)) & \text{if} \quad L > w > w^* \vee w^* > \tilde{w} > w \end{cases}$$

R's equilibrium strategy is illustrated in Figure 2 for a given $\tilde{w} > 0$, $w^* < L$, and $p^*(w^*) < L$. This figure points out that R's equilibrium strategy $p^*(w)$ jumps at $w = \tilde{w}$ as well as at $w = w^*$. Moreover, we see that $p^*(w) < p_L^M(w)$ at $w = w^*$.

Figure 2: Retailer's Equilibrium Strategy



[3] Employing $p^*(w^*) = L$ in (6) we get $w^* = L - 2(\alpha H + (1-\alpha)L - L) + 2\sqrt{(\alpha H + (1-\alpha)L - L)\alpha(H-L)} = L - 2\alpha(H-L) + 2\sqrt{\alpha^2(H-L)^2} = L$

5.2 Pooling Equilibrium

After having characterized R's equilibrium strategy, we show in this subsection that pooling equilibria can exist. To establish a pooling equilibrium, we consider the restrictive case where low (high) quality type can realize their monopoly profits when demand is low by charging $w^M_{L,L} = \frac{L}{2}$ ($w^M_{L,H} = \frac{L+c_H}{2}$). This means that $w^M_{L,L}$ as well as $w^M_{L,H}$ are smaller than $\tilde{w}$ (or larger than w^*). Thus, suppose that R reacts to the monopoly wholesale prices for low demand of both quality types with $p^M_L(w)$. If it holds that charging $w^M_{L,L}$ ($w^M_{L,H}$) generates less profit for low (high) quality type than when the equilibrium wholesale price w^* is charged, then a pooling equilibrium clearly exits as all other deviations would result only in a loss of margins. Hence, a pooling equilibrium exits if both is fulfilled

$$\Pi^*_L(p^*(w^*)) \geq \Pi^L_L(p^M_L(w^M_{L,L})) \tag{7}$$

$$\Pi^*_H(p^*(w^*)) \geq \Pi^L_H(p^M_L(w^M_{L,H})) \tag{8}$$

Setting expression (8) equal and subtracting it from (7), we can derive for $w^* < L$ following two thresholds

$$\overline{w} = \frac{L}{2} + \frac{c_H}{4} + \sqrt{\alpha(H-L)(L-\frac{c_H}{2})} \tag{9}$$

$$\underline{w} = \frac{L}{2} + \frac{c_H}{4} - \sqrt{\alpha(H-L)(L-\frac{c_H}{2})} \tag{10}$$

Now, we can establish a pooling equilibrium under vertical separation[4] if

$$w^* \in (\underline{w}, \overline{w}) \tag{11}$$

For a equilibrium wholesale price w^* larger or equal than L where $p^*(w^*) = w^*$ we have a less restrictive upper bound such that a pooling equilibrium

[4]Note, that by our assumption on R's reaction, this condition ignores all cases where $w^M_{L,i} \in [\tilde{w}, w^*]$. Then deviating to $w^M_{L,i}$ results only in a loss of margins as R doesn't react with a different price. So this condition restricts a pooling equilibrium to generate larger profits for both quality types than the profit maximizing wholesale price for low quality demand. Hence, there may also exist pooling equilibria for smaller or larger wholesale prices.

can exist. In this case we need to have $w^* < \alpha H + (1-\alpha)L + \frac{c_H}{8} - \frac{L}{4}$. This threshold is clearly larger than $\overline{w}$ from (9).

So far we have shown under which conditions on w^* a pooling equilibrium can be established. Still, we have to show when a equilibrium wholesale price w^* exists inside this interval. Thus, we denote w_L^* for low and w_H^* for high quality type as their profit maximizing wholesale price for the pooling demand $D^*(p^*(w^*))$. As for the same retail price demand is $D^*(p(w)) > D^L(p(w))$ and by $p^*(w^*) < p_L^M(w^*)$, we have that low (high) quality type makes higher profits when charging w_L^* (w_H^*) than $w_{L,L}^M$ ($w_{L,H}^M$). By $H > L$ and $\alpha \in (0,1)$ both prices are clearly larger than $w_{L,L}^M = \frac{L}{2}$. Setting $\underline{w}$ equal to $\frac{L}{2}$, we find that a pooling equilibrium with $w^* > \frac{L}{2}$ can exist for $c_H \in (0, L)$ if $\alpha > \frac{L}{8(H-L)}$.[5] Moreover, if $c_H \to 0$ we find that $\underline{w} < \frac{L}{2}$ holds without restrictions on α and H.

By $c_H > 0$ we have that $w_H^* > w_L^*$. So the upper bound $\overline{w}$ is relevant for high quality type. For $w^* \to L$ we have that $p^*(w^*) \to w^*$ which can be graphically observed in Figure 1. Then high quality type maximizes profits with $w_H^* = \frac{\alpha H + (1-\alpha) + c_H}{2}$ which needs to be smaller than $\alpha H + (1-\alpha)L + \frac{c_H}{8} - \frac{L}{4}$ such that a pooling equilibrium can sustain. This is the case if $\alpha > \frac{L}{4(H-L)}$ and $c_H \in (0, L)$. As $\frac{L}{4(H-L)} > \frac{L}{8(H-L)}$ this obviously also implies $\underline{w} < \frac{L}{2}$. Here we have again that if $c_H \to 0$ then $w_H^* < \overline{w}$ holds without restrictions on α and H.

In a nutshell, this subsection has provided conditions for a pooling equilibrium. We find that a pooling equilibrium exists either if α and H are sufficiently large or c_H is sufficiently small. Our restrictive assumptions in this subsection suggest that the here derived conditions on α and H can be loosened such that a pooling equilibrium may still exist. The whole parameter combinations of α and c_H that make a pooling equilibrium possible is numerically analyzed in section 6.

[5] In this case we also have $\overline{w} > L$.

5.3 Intuitive Criterion

In this subsection we analyze when a pooling equilibrium satisfies the Intuitive Criterion (IC). The IC by Cho and Kreps (1987) rules-out equilibria by requiring beliefs to be 'reasonable'. As the IC is usually applied to games where one player sends a signal which a second player receives, we first have to adopt the IC to our model where we face three players.

Suppose R observes a wholesale price $\check{w} \neq w^*$ and interprets that high quality type has charged this price. Then R optimizes its retail price according to the demand for high quality and charges $p_H^M(\check{w}) = \frac{H+\check{w}}{2}$. Observing such a price $p_H^M(\check{w})$ consumers belief that the product is of high quality and demand $D^H(p_H^M(\check{w})) = H - p_H^M(\check{w})$. We have to note that $p_H^M(w) = \frac{H+w}{2}$ is always larger than $p^*(w^*)$. So signaling quality through price is under vertical separation not only costly because of the price distortion but also through double marginalization.

A pooling equilibrium satisfies now the IC if it is *not* the case that high quality type wants to charge $\check{w}$ and low quality type doesn't want to mimic this deviation. More formally, a pooling equilibrium fails to satisfy the IC if there exists a wholesale price $\check{w} \in [0, H]$ for which *both* following conditions hold

$$\Pi_H^*(p^*(w^*)) \quad < \quad \Pi_H^H(p_H^M(\check{w})) \tag{12}$$

$$\Pi_L^*(p^*(w^*)) \quad \geq \quad \Pi_L^H(p_H^M(\check{w})) \tag{13}$$

In turn, if there exists *no* wholesale price $\check{w}$ such that *both* conditions (12) and (13) hold, then a pooling equilibrium satisfies the IC. So we need to have at least one of these two conditions to be violated to establish a pooling equilibrium that satisfies the IC. To show that a pooling equilibrium under vertical separation can satisfy the IC, we simply study when condition (12) fails. Then high type doesn't want to charge $\check{w}$. Disregarding if low quality type wants to mimic or not, a pooling equilibrium satisfies the IC if condition (12) fails. However, as this approach ignores that low type would want to mimic high type's deviation which violates (13), our analysis derives only restive conditions on pooling equilibria to satisfy the IC.

Charging $\check{w} = \frac{H+c_H}{2}$ maximizes high quality type's profit when both, R and consumers, believe that quality is high. Thus, high type will never want to deviate from w^* to $\check{w}$ if

$$\frac{(H-c_H)^2}{8} \quad < \quad (w^*-c_H)D^*(p^*(w^*)) \tag{14}$$

To analyze when (14) holds, we assume again for simplicity that w^* is in the left-neighborhood or larger than L. In such a case we have a retail price $p^*(w^*)$ equal to the equilibrium wholesale price w^*. Figure 1 shows that the larger H[6] is, the closer $p^*(w^*)$ shifts to w^*. At $w^* = L$ the equilibrium retail price $p^*(w^*)$ converges to w^* disregarding the level of H. Obviously, a equilibrium wholesale price $w^* \geq L$ exists, if α and H are sufficiently large.

Using this assumption to solve (14) for w^*, we can derive

$$w_{1,2}^{*IC} = \frac{\alpha H + (1-\alpha)L + c_H \pm \sqrt{(\alpha H + (1-\alpha)L - c_H)^2 - \frac{(H-c_H)^2}{2}}}{2}$$

A pooling equilibrium with a wholesale price $w^* \in (w_1^{*IC}, w_2^{*IC})$ satisfies the IC as then high quality type will never want to deviate. It can be easily shown that high type's pooling equilibrium profits maximizing wholesale price $w_H^* = \frac{\alpha H+(1-\alpha)L+c_H}{2}$ is inside this interval. Considering w_H^* as the unique wholesale price, we can specify (14) as follows

$$\frac{(H-c_H)^2}{8} \quad < \quad \frac{(\alpha H + (1-\alpha)L - c_H)^2}{4}$$

Irrespective of the level of H this holds for all $c_H \in (0, L)$ if $\alpha > \frac{1}{\sqrt{2}}$. For $c_H \to 0$ we need to have $\alpha > \frac{H-\sqrt{2}L}{\sqrt{2}(H-L)}$ which is smaller than $\frac{1}{\sqrt{2}}$. Hence, we find that any existing pooling equilibrium satisfies the IC if α is sufficiently large. Similar as in the subsection above, we have to note that by ignoring low type's role a pooling equilibrium may still satisfy the IC for a smaller α.

[6] This is the same for α.

5.4 Higher Profits

After having analyzed conditions under which a pooling equilibrium exists and further satisfies the IC, we now have to consider when such an equilibrium generates higher profits than when M decided to vertically integrate. Otherwise M would obviously not want to vertically separate. To do this, we compare the expected profits in the case of vertical separation with the case of vertical integration. The expected profits are given by $\alpha\Pi_H + (1-\alpha)\Pi_L$.

When M decided to vertically integrate and sells its product directly to consumers, the expected profit is for any parameters limited to $\frac{L^2}{4}$. This is the profit of low quality type. By $c_H > c_L = 0$ high quality type makes smaller profits. Consequently, the expected profits when M sells its product directly decrease in α. Due to this limitation, we can easily show that there exits a parameter space where M earns higher expected profits. Thus, we choose as equilibrium wholesale price again $w_H^* = \frac{\alpha H+(1-\alpha)L+c_H}{2}$ and make use of the same assumption as before that $w^* \geq L$.

In this case M generates higher expected profits when vertically separating if

$$\alpha\frac{(\alpha H+(1-\alpha)L-c_H)^2}{4}+(1-\alpha)\frac{(\alpha H+(1-\alpha)L)^2-c_H^2}{4}\geq\frac{L^2}{4}$$

For $c_H \in (0, L)$, we find that this holds if $\alpha \geq \frac{L}{H-L}$.[7] If this is the case then the condition for a pooling equilibrium derived above is clearly satisfied. Obviously a $\alpha < 1$ can only satisfy this if $H > 2L$.

Summarizing, this section shows that vertical separation can establish a pooling equilibrium that satisfies the IC *and* where the expected profit is larger than when M sells directly to consumers. This is the case if α and H are sufficiently large and c_H doesn't exceed L. As the here developed conditions for the existence of such a situation are rather restrictive, we have to note that larger parameter spaces may allow for higher expected profits under vertical separation.

[7] We derive this by solving $\Pi_H^*(w_H^*) > \frac{L^2}{4}$ for α. By $c_H > c_L = 0$ we have that $\Pi_L^*(w_H^*) > \Pi_H^*(w_H^*)$ which implies $\Pi_L^*(w_H^*) > \frac{L^2}{4}$.

6 Numerical Analysis

Since section 5 provides only an analytically proof that under vertical separation a pooling equilibrium which satisfies the IC and generates higher expected profits can exist, we simulate in this section our model numerically. The following numerical analysis allows us to show the parameter space where M wants to vertically separate.

Examining our numerical analysis we first calculate for each parameter combination if under vertical separation a pooling equilibrium exists. Thus, we compute for each pooling equilibrium price w^* the best deviation profits for high as well as low type. Doing this we get two $[0, H] \times [0, H]$ matrices. Due to restrictions in computing power, we have to restrict for these matrices the accuracy to three digits.[8]

In the next two steps of our numerical analysis, we investigate if an existing pooling equilibrium satisfies the IC and if it generates higher expected profits than the separating equilibrium outcome when M decided to sell its product directly to consumers. As we need for these two steps only a vector of the length H, we have no such strict restrictions as for the deviation matrices above. To avoid errors due to inaccuracy, we set for this vector the number of digits to five.

When analyzing if an existing pooling equilibrium satisfies the IC and if it generates higher profits than under vertical integration, we need to fix for each level of H and each parameter combination of α and c_H an equilibrium wholesale price w^*. Clearly, an equilibrium wholesale price w^* where high quality type makes a large and low quality type makes small profits, is more likely to satisfy the IC. So regarding this, we have two candidates for choosing our w^*. First, the pooling equilibrium wholesale price w^* that maximizes high type's profits which we define for the following as w^*_{maxH}.[9] Second, the pooling equilibrium wholesale price w^* that minimizes low type's profits which we define as w^*_{minL}.

[8] Under this restrictions we have two matrices with H^2-million entries

[9] Note that w^*_{maxH} maximizes high quality type's profit among all possible pooling equilibria and not as w^*_H for the pooling demand. So both prices are only equal if under w^*_H a pooling equilibrium exits.

For our purpose, we choose $w^* = w^*_{maxH}$ to maximize the chances that (12) fails which allows a pooling equilibrium satisfying the IC. Obviously, by giving the highest possible pooling equilibrium profit, w^*_{maxH} minimizes high quality type's incentive to deviate from the pooling equilibrium price. Further, such a pooling equilibrium wholesale price w^*_{maxH} does not generate the highest pooling equilibrium profit for low quality type as by $c_H > 0$ we have $\frac{\partial \Pi^*_L(w^*_{maxH})}{\partial w} < 0$.[10] As an increase in c_H shifts w^*_{maxH} upwards but affects low type's profits only through the equilibrium wholesale price, we have that w^*_{maxH} even minimizes low quality type's profit among all existing pooling equilibria if c_H is sufficiently large. So beyond minimizing high type's incentive to deviate, choosing w^*_H evolves also incentives for low quality type to mimic high type's deviation. Consequently, fixing the equilibrium wholesale price to $w^* = w^*_{maxH}$ can also violate condition (13).

In addition to optimizing the chance that a pooling equilibrium satisfies the IC, choosing w^*_H instead of w^*_{minL} maximizes the chance that the pooling equilibrium generates higher expected profits than the separating equilibrium when M decided to vertically integrate. As w^*_{minL} is always a corner point among all pooling equilibrium wholesale prices w^*,[11] choosing w^*_{maxH} generates higher expected profits, $\alpha\Pi^*_H(w^*) + (1-\alpha)\Pi^*_L(w^*)$, than w^*_{minL} for all $w^*_{maxH} \neq w^*_{minL}$. Hence, we apply for our numerical analysis w^*_{maxH}.

6.1 Numerical Results

The following three figures illustrate our numerical analysis. Employing w^*_{maxH} they show under which α and c_H a pooling equilibrium exists, if it satisfies the IC, and if it generates higher expected profits than when M decided not to vertically separate.[12] The three figures vary among the level

[10] This is of course only the case if we have more than one wholesale price under which a pooling equilibrium exists.

[11] See Proof A1 in the Appendix

[12] Comparing to the Figures 6, 7, and 8 in the appendix where w^*_{minL} is employed, we find support for our theoretical considerations on choosing $w^* = w^*_{maxH}$. First, the parameter space that satisfies the IC is significantly larger when using w^*_{maxH}. Second, in contrast to w^*_{maxH} choosing w^*_{minL} never generates higher profits than under vertical integration.

of demand for high quality H. The demand for low quality is normalized to one, $L = 1$. The parameters α and c_H are limited to $(0, 1)$. In Figures 3, 4, and 5 a pooling equilibrium exists under the light grey areas. The dark grey areas depict the parameter spaces where a pooling equilibrium satisfies the IC. The vertical lines indicate areas where M generates higher expected profits when it decided to vertically separate than when selling directly without R.

Figure 3: Moderate setting: $H = 2, L = 1$

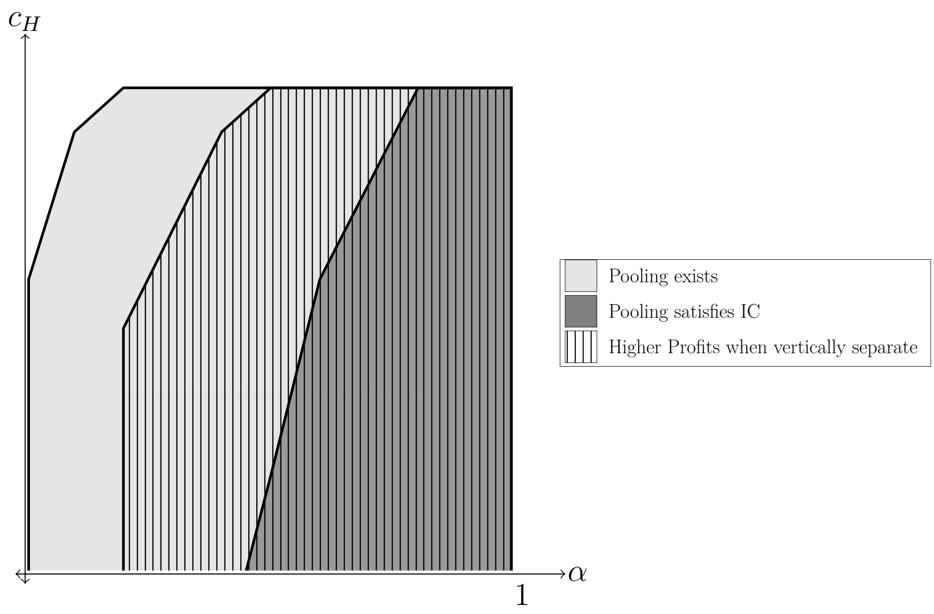
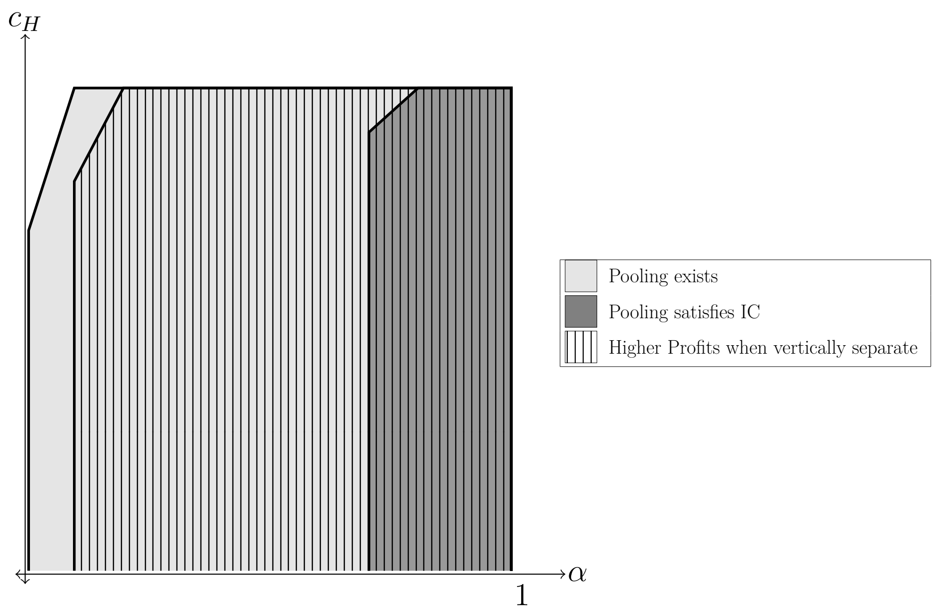


Figure 3 uses a 'moderate' demand for high quality where $H = 2L$. Compared to the 'high' and 'low' settings this one contains the largest parameter space of α and c_H where a pooling equilibrium satisfies the IC *and* generates higher expected profits than when M decided not to vertically separate. In line with our analytical findings, Figure 3 clearly shows that a larger α leads to higher expected profits when M vertically separates compared when M vertically integrates. The reason for this finding is, that profits in the case of vertical integration are limited to $\frac{L^2}{4}$ for both quality types regardless of the level of α. The profits in the case of vertical separation, however, increase in α as the demand in the pooling equilibrium is $D^*(p^*(w^*)) = \alpha H + (1-\alpha)L - p^*(w^*)$ and $H > L$. Further we see that a pooling equilibrium satisfies the IC if α is large. As derived in subsection 5.3 we find that a pooling equilibrium satisfies the IC for all $c_H < L$ if $\alpha > \frac{1}{\sqrt{2}}$. In addition we see when $c_H \to 0$ that $\alpha > \frac{H-\sqrt{2}L}{\sqrt{2}(H-L)}$ (under this setting $\alpha > 0.41$) is sufficient such that an existing pooling equilibrium satisfies the IC.

Figure 4: High setting: $H = 4, L = 1$



A 'high' demand for high quality, $H = 4L$, is considered in Figure 4. As a larger demand for high quality increases the pooling equilibrium profits, compared to the other two settings this figure expectably reveals that a high demand generates the largest area under α and c_H where a pooling equilibrium exists. Moreover, under this setting vertical separation yields already for a small[13] α to higher expected profits than vertical integration. Compared to the other two settings, high demand clearly generates higher profits in larger parameter spaces of α and c_H. This rather unsurprisingly finding

[13]Starting from $\alpha = 0.09$ expected profits are higher. This is substantially smaller than the threshold derived in 5.4 which is for this setting $\alpha > \frac{1}{3}$.

can be explained in the same way as above, that higher expected profits occur when α increases. While a larger demand for high quality H increases in a pooling equilibrium, separation equilibrium profits are by construction for both quality types limited to $\frac{L^2}{4}$. Thus, separation equilibrium profits do not depend on the level of H. Now, as profits increase in H when M decided to vertically separate but not when M decided to vertically integrate, we have that a high H results in increasing incentive to vertical separation. Both, large demand for high quality H and high probability to have high quality α lead to higher expected profits under vertical separation.

In addition Figure 4 shows, that the area under which a pooling equilibrium satisfies the IC is significantly smaller than for the 'moderate' demand setting in Figure 3 and 'low' demand setting in Figure 5. Obviously a larger demand for high quality induces higher incentives for high quality type to deviate by charging a higher wholesale price.

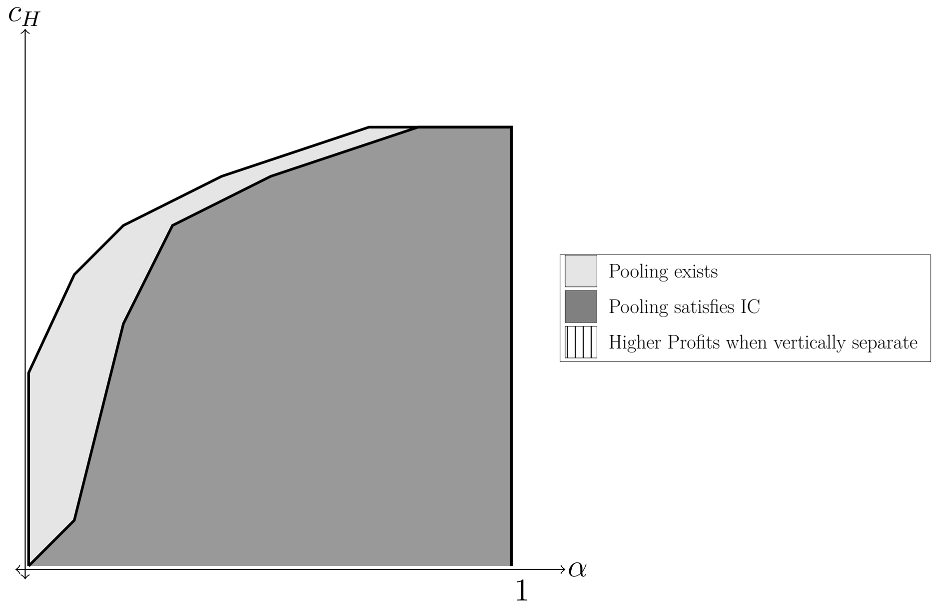
Figure 5 applies a 'low' demand setting for high quality. Here we set $H = 1.1L$. Two characteristics of this setting are noticeable. First, among all three settings this one has the smallest area where a pooling equilibrium exists. Second, from Figure 5 we can observe that for such a low demand the largest area of pooling equilibria satisfies the IC. However, in striking contrast to the two other settings, no parameter combination of α and c_H generates higher expected profits when M decided to vertically separate than under vertical integration. So according to our analytical findings we observe that if the difference between demand for high quality and demand for low quality, $\Delta = H - L$, is small, M will never want to vertically separate.

In general, if $H \to L$ then in the case of vertical integration M charges $\frac{L+c_i}{2} \quad \forall i = \{low, high\}$. This brings profits of $\frac{(L-c_i)^2}{4}$ which is in this case the maximum each quality type can gain. However, profits are by double marginalization always strictly smaller under vertical separation than under vertical integration.[14]

Summarizing our numerical findings, we find that vertical separation allows under certain parameters a pooling equilibrium that satisfies the IC *and* that generates higher expected profits than under vertical integration where

[14] See Proof A2 in the Appendix A.

Figure 5: Low setting: $H = 1.1, L = 1$



firm signals quality through prices. So the findings of the numerical analysis support our thesis, that for certain parameter settings M can overcome the inefficient outcome of a separating equilibrium through vertical separation. By our restrictive assumptions these parameter spaces are clearly larger than theoretically derived in section 5.

Moreover, the numerical analysis reveals a 'trade-off' between the characteristics that a pooling equilibrium under vertical separation satisfies the IC and that it generates higher expected profits than without vertical separation. Comparing Figures 4 and 5 demonstrates that on the one hand under a high demand setting a larger parameter space of α and c_H generates higher profits than without vertical separation, while a low demand setting can lead to a small or as in our case no area where profits are larger. If $H \to L$ we find – numerically as well as analytically – that vertical separation is never profitable. On the other hand, we observe that more parameter combinations allow a pooling equilibrium that satisfies the IC when demand for high quality is low.

7 Conclusion

Signaling quality through prices generates inefficiencies when a monopoly manufacturer sells its product directly to uninformed consumers. In this case a separating equilibrium occurs where price distortions result in smaller profits. This outcome is the only one when requiring beliefs to be 'plausible'. However, when distributing the product through a retailer one can overcome such an inefficient outcome.

In this paper we have studied a model with linear demand where a manufacturer sells its product through a retailer. Our results demonstrate that under vertical separation a pooling equilibrium can exist which satisfies the Intuitive Criterion – that means that beliefs have to be plausible – *and* generates larger profits. Vertical separation features, first, incomplete control over and, second, higher costs of signaling quality through prices. Thus, under vertical separation a manufacturer can credibly hide the knowledge of quality. This advantage can't be imitated by vertical integration. Moreover, larger profits overweight the costs of double marginalization through the retailer. Finally, this paper highlights a novel strategic motive why firms engage in vertical separation.

The present paper only provides an analytical proof that vertical separation can satisfy the Intuitive Criterion and generate higher profits than vertical integration. Hence, clear conditions under which this is the case should be of considerable interest of future research. Extending our model by including a fraction of informed consumer and repeating the game would be further worthwhile to study.

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Appendix A

Proof A1. Let w^*_{minL} be no corner point of all w^* and w^*_{maxL} be the value of all w^* that maximizes low type's profit. Then by definition of w^*_L we have either $w^* \leq w^*_{maxL}$ or $w^* \geq w^*_{maxL}$. In the case of $w^* \geq w^*_{maxL}$ we have by linear demand which is decreasing in w^* and as w^* is no corner point, that there exists a value $w^*_L + \varepsilon$ that generates smaller profits than w^*_{minL}.

In the case of $w^* \leq w^*_{maxL}$ we have by the margins which are increasing in w^* and as w^* is no corner point, that there exists a value $w^*_{minL} - \varepsilon$ that generates smaller profits than w^*_L.

Taking both cases together w^*_{minL} doesn't minimize low type's pooling equilibrium profits which contradicts the definition of w^*_L. ↯ □

Proof A2. Setting $H = L$ and maximizing the expected profits when M decided to vertically separate, gives $\frac{(L-\alpha c_H)^2}{8}$ which is clearly smaller than the expected pooling equilibrium profit without vertical separation, that is$\frac{(L-\alpha c_H)^2}{4}$. Setting this now smaller than the expected separation profit without vertical separation, $\alpha\frac{(L-c_H)^2}{4} + (1-\alpha)\frac{L^2}{4}$, we get $\alpha < 1$ which is satisfied by our assumption $\alpha \in (0, 1)$. □

Figure 6: Moderate setting: $H = 2, L = 1$

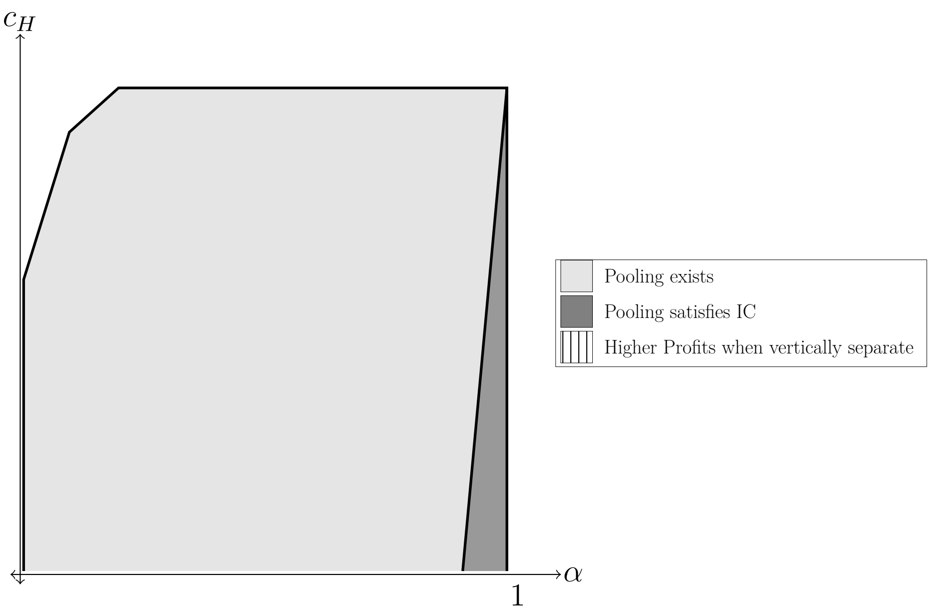


Figure 7: High setting: $H = 4, L = 1$

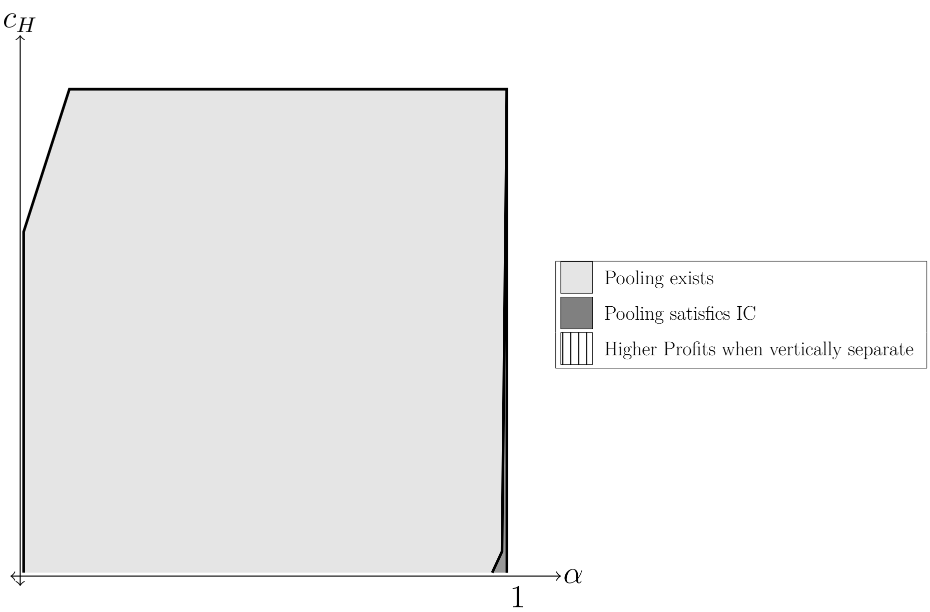
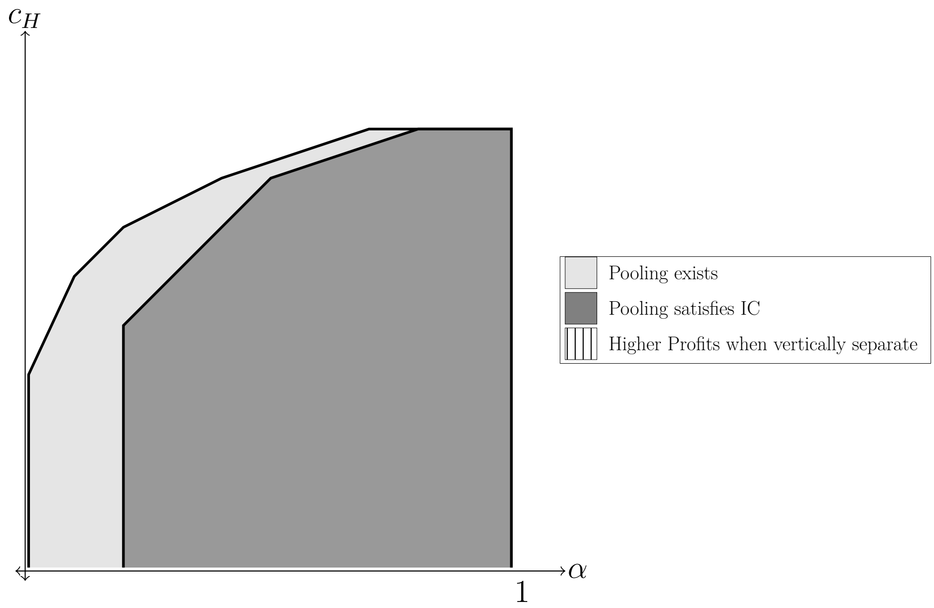


Figure 8: Low setting: $H = 1.1, L = 1$



Appendix B

Zusammenfassung

In den meisten Industrien beobachten wir vertikale Strukturen wo HändlerInnen die Produkte von ProduzentInnen an KonsumentInnen verkaufen. Ziel dieser Arbeit ist die Erstellung eines Modells, das die Vorteile einer Separierung aufzeigt, die nicht durch vertikale Integration imitiert werden können. Wenn eine monopolistische ProduzentIn die einzige AkteurIn ist, die die Qualität ihres Produktes kennt und diese direkt an die KonsumentInnen vertreibt, dann erfüllt nur ein separierendes Gleichgewicht, wo Preise die Qualität signalisieren, das Intuitive Criterion (IC). Vertikale Separierung als eine glaubhafte Möglichkeit das Wissen über die Produktqualität zu verstecken, kann ein vereinigtes Gleichgewicht ermöglichen, das das IC erfüllt und höhere Profite generiert. Daher erlauben wir der ProduzentIn das Produkt entweder an eine HändlerIn oder direkt an die KonsumentInnen zu verkaufen.