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„Money Creation, Debt Dynamics and Financial Instability“

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Abstract

This thesis surveys recent contributions on the macroeconomics of private debt. Particular focus is given on two main channels which might lead to recessions or even depressions. On the one hand, there is an aggregate demand channel: indebted households cut down their consumption expenditures and repay their outstanding debt in response to a macroeconomic shock to the economy-wide safe level of debt. Monetary policy might end up in a liquidity trap if the deleveraging process is large enough, leading to severe recessions. On the other hand, there is an aggregate supply channel through financial intermediaries’ portfolio decisions. Intermediaries’ balance sheets are affected by macroeconomic shocks leading to failing of some investment projects financed via debt. As intermediaries contract their balance sheets, deflation might arise amplifying the initial shock and leading to further reductions of investment financed by intermediaries. In light of recent empirical work, both channels seem to have contributed substantially to the recent financial and economic crisis.
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I Introduction

“[W]hat happens in our economy is so largely determined by financial considerations, economic theory can be relevant only if finance is integrated into the structure of the theory.”


If there is one thing the current economic crisis managed to accomplish, then it is that it revived economists’ interest in understanding money, credit and financial intermediaries. Especially public sector debt drew considerable attention, while private sector debt was largely neglected in the immediate aftermath of the crisis. According to Roxburgh et al. (2010) total debt had been markedly increasing before the outbreak of the crisis, and afterwards entered a phase of aggregate debt reduction. Roxburgh et al. (2012) note that financial crises are typically followed by episodes of deleveraging and resultant hampered growth. They identify two phases of deleveraging. First the private sector, including households, firms and financial institutions, reduces its debt level significantly, economic growth slows down or turns negative and public debt rises. In the second phase, growth picks up and public debt gradually recovers. In historical terms, the whole process of deleveraging might last more than a decade on average until output growth fully recovers.

Interest in economic thinking about money and debt has varied substantially over time (see e.g. Freixas and Rochet, 2008, ch.6). For instance, Bernanke (1983) argues that high cost and low availability of credit during the Great Depression reinforced the depressing effects on aggregate demand. In addition, he points out the unprecedented creation of debt in the preceding decade and that deflation caused wide-spread bankruptcy.

Most prominently, Fisher (1933) developed the theory of debt-deflation in which falling prices due to debt liquidation increase the real burden of debt, which is followed by further deflation. If the economy is in a state of overindebtedness, this will eventually lead to liquidation of debt, and subsequent falling prices. This negatively affects the net worth of the private sector and leads to a reduction in output, loss of confidence and a fall in the nominal interest rate. This puts an additional strain on the deleveraging process and might lead to further selling of assets. Put differently, the repayment of debt may have contractionary effects on money supply and the price level.

Fisherian debt-deflation has been mostly overlooked in the past century,1 with notable exceptions being Tobin (1980), Minsky (1982) and Kindleber-

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1One popular explanation might be that “[…] debt-deflation represented no more than a redistribution from one group (debtors) to another (creditors)” and “[a]bsent implausibly large differences in marginal spending propensities among the groups […] pure redistribution should have no significant macroeconomic effects” (Bernanke, 1995, p. 17).
Especially Minsky (1986) – whose work gained popularity recently – famously argues that financial crises are endogenously created in modern economies. Calm periods of economic volatility lead to complacency about debt and foster rising leverage and risk-taking among firms and households. The banking sector might be willing to accommodate to the desire for leverage. A resulting credit boom eventually leads to unsustainable levels and subsequent busts as well as debt-deflationary spirals. In other words, economic stability itself creates instability, which will ultimately culminate in a recession or a depression once economic conditions worsen. Moreover, Minsky’s work strongly emphasises the role of financial intermediaries in the process of money and debt creation.

Recent contributions suggest that there exists a causal link between debt, financial instability and business cycle dynamics. This thesis aims to address the following research question: what role do private debt and financial intermediaries play in financial crises? To answer this question, a particular focus is given to relevant macroeconomic models, which incorporate debt-deflation à la Fisher and financial instability à la Minsky.

Section II revisits the textbook approach to debt and financial intermediaries and contrasts it with a relatively new approach to money creation. Given this new approach, the necessity to study the financial sector for macroeconomic issues becomes evident. Section III provides a brief literature survey reviewing important contributions centring around debt and financial intermediaries in macroeconomic models. A DSGE-type model of dynamic deleveraging is presented in section IV. Simulations of a linearised version of the model are able to replicate Minsky-type results qualitatively. Section V provides a novel approach to macroeconomic modelling based on continuous-time optimal portfolio choice, also featuring financial instability and debt-deflation. The exposition of both models is completed by a brief discussion of their main insights, policy implications and possible shortcomings in section VI. Section VII concludes, including suggestions for further research questions that build on the aforementioned approaches.

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2See von Peter (2005) for a detailed discussion of Fisherian debt-deflation.
II A Primer on Money Creation

Following Blanchard (2009), introductory textbook economic theory tells a simple story of banks acting as intermediaries between lenders and borrowers. Banks collect deposits from savers and lend them out to borrowers, such as households who take out mortgages to buy houses, or firms looking to finance their investments. Bank lending is restricted by the reserve requirement set by the central bank, which ensures that each bank has sufficient liquidity when deposits are withdrawn. Reserves are partly held in cash and partly in an account at the central bank. In this simplified setup, liabilities of banks are deposits and their assets consist of reserves and loans. Liabilities of the central bank, on the other hand, are the reserves that banks deposit in their accounts at the central bank and the currency in circulation. The lending capacity of banks is ultimately constrained by the supply of central bank money (either in the form of currency or reserves). By increasing the supply of central bank money (i.e. high-powered money), the central bank can stimulate the loan supply of banks by increasing available deposits. The overall increase in the level of money in the economy depends on how much of this high-powered money is deposited and lent out by banks. These credits, in turn, create more deposits that can be lent out. After several rounds of passing on available deposits, the overall supply of money is equal to a multiple of the initial injection of central bank money. This process of money creation involves a money multiplier, which is simply the inverse of the reserve requirement. This ideal notion of a financial sector acting as a frictionless intermediary clearly excludes the possibility of financial crises.

In spring of this year, the Bank of England publically distanced itself from the conventional theory of money creation (McLeay, Radia and Thoma, 2014). Viewing banks as mere intermediaries ignores the fact that banks are actually creators of money. In contrast to conventional theory, banks create money in the sense that they determine the overall volume of debt and deposits in the economy.

To elaborate on the mechanism of money creation, consider the money balance sheets of different sectors of a stylised economy as in figure 1. The central bank holds some non-money assets (e.g. government debt) and has the reserves of commercial banks and the currency in circulation on its liability side (where the latter two constitute the available base money). Balance sheets of commercial banks are composed of reserves and currency (to meet deposit withdrawals) on their asset side and deposits of the household sector as liabilities. The household sector represents the private sector of con-

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3Bundesbank (2014) basically states the same in a recent version of their schoolbook on monetary policy.

4Commercial banks balance sheets usually also include non-money assets such as gov-
sumers and companies. Its balance sheet only shows broad money assets (i.e.
deposits and currency holdings) and corresponding non-money liabilities.\textsuperscript{5}
Now, suppose an agent from the household sector approaches a commercial
bank for a loan. By granting a new loan, the affected commercial bank in-
creases its asset holdings while the agent increases its liabilities by the same
amount. By either spending or investing the available amount of additional
money, some other agents are able to increase their deposit holdings – poss-
sibly at some other bank. Ultimately, both sides of the aggregate balance
sheets of the households and the commercial banks increase, thus creating
new broad money while leaving the base money unchanged. Now, there is
some bank in the economy with a surplus of deposits, while another bank
might face a shortage of funds to meet some regulatory requirement. The
bank in the latter case can now either receive a loan from another bank
or acquire additional funds from the central bank, while the surplus bank
might create additional loans, thus further increasing the supply of available
deposits in the banking sector.\textsuperscript{6}

McLeay, Radia and Thoma (2014) emphasise that the quantity of base
money does not directly constrain the volume of bank lending. A higher
stock of deposits might force banks to increase their reserves at the central
bank. In normal times,\textsuperscript{7} central bank money needed for reserves can be
acquired on demand. Even though banks create money through their lending
behaviour, they cannot do so without limits as indicated by Tobin (1963).
Broadly put, there are three main restrictions on money creation:

(i) \textbf{Behaviour of households:} The non-bank private sector might use
additional loans to expand their consumption or investment expendi-
tures, which eventually leads to the passing on of newly created money
and possibly inflationary tendencies. However, households might also
take out loans to repay outstanding debt, thereby effectively destroying
money. Thus, shifts in expectations, preferences or beliefs may have
an impact on the overall money in the economy.

(ii) \textbf{Constraints of banking:} Even though the banking sector as a whole
can create loans with matching deposits on its aggregate balance sheet,
individual banks cannot do so without limitations. They have to ade-
uately manage risk associated with loans, which involves both liquidity risk
(holding a sufficient amount of deposits, reserves, currency or
other liquid assets to be able to repay all of their depositors) and credit

\textsuperscript{5}This illustration excludes real assets such as housing or non-money liabilities including
existing secured or unsecured loans.
\textsuperscript{6}For ease of exposition, this illustration completely abstracts from lending restrictions
of individual banks and lending between financial institutions of different types, which is
undeniably a crucial feature of the financial sector in reality.
\textsuperscript{7}The term ‘normal times’ refers to non-crisis periods.
Figure 1: Money creation illustrated in sectoral balance sheets.
Source: Adapted from McLeay, Radia and Thoma (2014).
risk (accounting the possibility of defaulting debtors). In order to re-
maintain profitable, market forces should ensure that banks adequately
manage the risks associated with granting new loans. Prudential reg-
ulation usually aims to discourage banks from building-up excessive
risk. The interest rate set by the monetary authority serves as the
ultimate constraint on money creation.

(iii) Monetary policy: Nowadays, many central banks typically focus on
setting the price for money (the interest rate) rather than choosing
the quantity of available reserves. Due to demand for central bank
money by commercial banks, changes in the central bank interest rate
have an impact on other interest rates in the economy. This means
that monetary policy usually affects the price of credit and the overall
volume of debt in the economy. In addition, some central banks adopt
more unconventional measures to increase financial market liquidity,
such as purchasing assets from distressed institutions or issuing bonds.

Before proceeding, it is useful to introduce and explain the notions of outside
and inside money, which will reappear in subsequent sections. Outside
money is money outside the private sector and is either backed by some
asset (e.g. gold) or fiat money (i.e. unbacked). As this includes currency in
circulation and bank reserves, it can be set equal to base money in the above
exposition. Inside money, on the other hand, is backed by some asset form
inside the private sector. This includes deposits and loans created through
the bank lending process from above. This means that inside money can be
equated to broad money minus base money.

In its essence, this thesis deals with the three aspects of money creation out-
lined above. As behaviour of households determines how much debt they
want to hold, the model discussed in section IV deals with the macroeco-
nomic effects of deleveraging. Households delever when they wish to reduce
their debt levels, which leads to a large scale reduction of inside money. As
this model abstracts from any explicit formulation of the financial sector, it
is sensible to look at a model where the health of the financial sector and the
ability to mitigate economic risk associated with debt foster the creation of
inside money. This is the crucial feature of the model presented in section
V. Conventional monetary policy is incorporated in both models. However,
limitations of monetary policy as a stabilising factor for the economy are
featured in both modelling approaches.

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8See e.g. Lagos (2006) for a thorough discussion of inside and outside money.
III Literature Review

There is an abundance of literature on the link between financial frictions as key drivers of macroeconomic fluctuations, that continues to increase in light of the ongoing recession. In frictionless economies, the distribution of debt has no effect. Differences in productivity, patience or risk-taking affect the flow of funds, but for aggregate outcome only total capital and total labour matter. In an economy with frictions, financial intermediaries are crucial in mitigating some of these frictions. By creating inside money, banks may overcome financial frictions and channel funds from less productive (or more patient) to more productive (or less patient) agents, but they might also generate additional fragility through potentially erratic money creation. Without a financial sector, funds are constrained by the available outside money. (Brunnermeier, Eisenbach and Sannikov, 2012)

This section provides a short review of relevant contributions on debt dynamics and financial frictions, as well as on the explicit modelling of commercial banks and inside money in macroeconomic models. As the survey is far from being complete, it is best to refer to Gertler and Kiyotaki (2010), Quadrini (2011) and Brunnermeier, Eisenbach and Sannikov (2012) amongst others for extensive discussions of the related literature.

A. Financial Frictions

Seminal contributions typically focus on the role of financial frictions and their implications for the dynamics of debt in macroeconomic models.

Bernanke and Gertler (1989) (BG) analyse the role of borrowers’ balance sheets in an overlapping generations model with asymmetric information. There are two classes of agents in the economy who live for two periods: lenders and entrepreneurs, the latter of which can create new capital goods that are used by firms to produce consumption goods. Entrepreneurs invest using their own wealth and borrow from lenders. The key friction of the model is the assumption of costly state verification; lenders face monitoring costs, as only entrepreneurs can see the results of their investment projects. A negative productivity decreases wages and thus entrepreneurs’ net worth. This tightens borrowing constraints and has a depressing effect on investment in capital. A decrease in capital reduces next period’s wage levels which implies lower net worth and a further drop in investment. Temporary shocks can therefore have persistent effects on economic activity.

Kiyotaki and Moore (1997) (KM) construct a model in which credit constraints arise from real estate serving both as a factor of production as well as collateral for loans. Their economy is populated by two types of agents: less productive and less patient lenders and more productive and more impatient borrowers. Borrowers have to secure their loans with real estate in incomplete contracts with lenders. If some highly leveraged firms are hit by
productivity shocks, they are forced to cut back on investment expenditure, including real estate. Falling asset prices cause the net worth of other highly leveraged firms to fall. As a result, asset demand of constrained firms continues to decline which causes asset prices to fall even further. Borrowers’ credit limits are affected by the prices of collateralised assets, while prices are in turn affected by the credit limit. Adverse feedback loops through asset prices cause initial small shocks to have potentially large effects.

Bernanke, Gertler and Gilchrist (1999) (BGG) consider three types of agents in a New Keynesian framework: risk-averse households, risk-neutral entrepreneurs and retailers facing staggered prices. Entrepreneurs are the only ones who hold capital used in production of the consumption good and serve as financial intermediaries. The production sector consists of heterogeneous firms in a perfectly competitive environment who use capital and labour to produce consumption goods and face random foreclosure. As new firms vanish and emerge constantly, they cannot accumulate enough wealth to be independent of financial intermediaries. Retailers buy consumption goods and and offer differentiated products in a monopolistically competitive environment. BGG introduce non-linear adjustment costs of capital, such that when a negative shock hits net worth, the price of capital falls. A lower price of capital further diminishes net worth, amplifying the initial shock. Similar to BG, endogenous variations in the balance sheets of firms are amplified by financial market conditions – a phenomenon the authors denote as financial accelerator.

Both KM and BGG consider the dynamic propagation and amplification of small shocks through asset prices. Furthermore, both approaches focus on linear approximations around the deterministic steady-state. However, it is important to emphasise that temporary shocks affecting net worth can have persistent and possibly large effects on economic activity.

For instance Kocherlakota (2000) constructs a simple model with credit constraints, building on a neoclassical growth model. Productive agents face credit constraints on how much they may borrow at an exogenously set interest rate. The agents use capital and land to produce output. Unanticipated increases in income have no impact on output as firms production is already considered to be optimal, so additional income is either consumed or saved. If large enough, negative income shocks may reduce output as credit-constrained entrepreneurs have to reduce their productive assets, thereby driving down asset prices, which, in its turn, shrinks debt capacity and depresses output. Thus, due to credit constraints, temporary income shocks may have asymmetric persistent effects.

Eggertsson and Krugman (2012) present a simple Keynesian-style model of debt-driven slumps with an endogenous natural rate of interest. They start out with a flexible-price endowment setting in which impatient agents borrowing from more patient agents face a debt limit. If the debt limit is reduced, impatient agents are forced to deleverage and cut spending. If the
deleveraging effect is large enough, the economy might end up at the zero lower bound (ZLB). If debt is contracted in nominal terms while the debt limit is defined in real terms, Fisherian debt-deflation arises, as a drop in the price level leads to a further decrease of the natural rate of interest, making the price level decline further.

Gu et al. (2013) follow a different methodology where they assume limited commitment as the key friction. Borrowers produce an intermediate good and promise to pay back lenders when their investments pay out. In order to avoid punishment by being excluded from future credit, borrowers honour their debt. However, they might choose to abscond from paying back. As lenders anticipate limited commitment, debt limits are endogenous. Gu et al. (2013) derive multiple equilibria which can lead to deterministic, chaotic or stochastic cycles through shifts in beliefs. If agents believe tomorrow’s debt limit will fall, they are more willing to pay back debt today to avoid being excluded from the credit market. This alleviates today’s endogenous debt limit. However, tomorrow, they expect the next day’s debt limit to go back up, in which case they prefer not to pay back debt, driving debt limits up. Depending on the magnitude of the shifts in beliefs, the resulting oscillations might lead to chaotic dynamics or sunspot equilibria.

Brunnermeier and Sannikov (2014a) build upon the work of BGG, KM and Kocherlakota (2000) in a continuous-time model with productive experts and less productive households. Due to financial frictions the distribution of wealth is crucial as experts are constrained when it comes to buying capital and using it productively. They derive significant non-linearities around the steady-state and uncover the volatility paradox: “the idea that the system is prone to crises even if exogenous risk is low” (Brunnermeier and Sannikov, 2014a, p.5). Novel features of this approach are the non-linear and asymmetric reactions of the system to shocks, i.e. positive shocks may be subject to little amplification, whereas negative shocks may lead to significant crisis periods and sluggish recovery phases.

B. Financial Intermediaries

As can be seen from above, the majority of seminal contributions on models of financial frictions are constructed without explicit modelling of a commercial banking sector. That is, financial contracts are arranged directly between agents without any special intermediation by other agents. Some papers, however, explicitly model banking.

Building on BG and BGG, Goodfriend and McCallum (2007) provide a two sector model with a goods-producing sector and a banking sector. Households own and work in both sectors, buy consumption goods and hold money for transaction purposes. The banking sector uses a Cobb-Douglas technology to produce loans (and hence deposits) by combining collateral and loan monitoring, where the latter is performed by households supplying
labour to banks. Government bonds and capital serve as collateral, but the latter is assumed to be inferior in the sense that the monitoring cost of capital are higher than for government bonds. As loans and deposits are costly to produce, as they require work effort, there are several different interest rates in the model. The banking sector generates two opposite effects: the banking attenuator dampens the effects of monetary policy shocks as the external finance premium is pro-cyclical. A positive monetary shock has a positive effect on consumption in the model. Higher consumption leads to an increase in deposit demand which must be met by an increase in loan production. This also requires an increase in monitoring effort and by concavity of the production function, the monitoring costs grow faster than the loans. Higher costs of lending increase the spread between loan and policy rate and dampen the effect on consumption. The second effect, the banking accelerator, stems from the positive effect of expansionary monetary policy on the marginal product of capital and the price of capital. This increase in the price of capital increases collateral value and, by the form of the production function, monitoring cost. So the increase in deposit demand is partly offset by a higher cost of loan production.

Iacoviello (2005) builds on BGG and KM and features an economy populated by patient households, impatient entrepreneurs, retailers and a central bank. Households consume, work and demand real estate and money. Entrepreneurs consume, hire households and use collateralised real estate to produce a homogeneous intermediate good. Retailers purchase the intermediate good and transform it into a final consumption good on a monopolistically competitive market which is the source of nominal rigidity. Negative monetary shocks drive up the real interest rate, discourage current consumption and depress output. As house prices fall, borrowing decreases as does investment, which reinforces the initial effect. In addition, deflation raises the cost of debt-service which leads to further declines in consumption and investment. Iacoviello (2014) closely follows a flexible price version of this model and introduces a banking sector. In this model, bankers maximise their private consumption by acting as intermediaries between households and entrepreneurs while facing regulatory constraints on the leverage of their balance sheets. As the production sector is bank-dependant for its operations, losses sustained by the banking sector can have substantial effects on the business cycle.

Kobayashi (2008) develops a New Keynesian model with financial intermediaries and studies the imperfect pass-through from money-market rates to lending rates. The model economy consists of intermediate-good and final-good producing firms, commercial banks, households and a central bank. Employers of the intermediate-goods sector have to pay wages in advance at the beginning of each period while receiving revenue at the end of each period. To compensate for the wage bills, firms borrow funds from commercial banks on a geographically segmented loan market which leads
Literature Review

to heterogeneity in lending rates. Commercial banks face Calvo-type pricing with respect to the determination of the loan rate. By incorporating profit-maximising behaviour of commercial banks, the difference between the policy rate and the retail lending rate becomes endogenous.

Similarly, Cúrdia and Woodford (2009) introduce a financial intermediary sector into a New Keynesian setting. Their model is populated by two types of consumers facing random preference shocks. There are agents with high marginal utility of consumption (borrowers) and agents with lower marginal utility of consumption (savers). Agents can only borrow from and lend to financial intermediaries who face some intermediation costs, as some agents might abscond from paying back loans. These costs determine the spread between loan and deposit rate. The cost and spread vary with the amount of lending.

Adrian and Shin (2010a) build a New Keynesian model in which they focus on the balance sheet activities of financial intermediaries as a determinant of the price of risk. Expanding balance sheets lead to an increase in the risk-bearing capacity of intermediaries and a fall in risk premia. Balance sheet quantities are indicative for the risk-taking capacity of the whole financial sector, and should therefore be taken into consideration by the monetary authority.

Eggertsson and Krugman (2012) also embed the deleveraging mechanism described above in a sticky-price New Keynesian type model where the deleveraging shock affects output instead of prices. In the appendix of their paper, they show how dynamic deleveraging can be modelled in a dynamic model with a banking sector, drawing on Cúrdia and Woodford (2009). Banks lend out available deposits in a loan-contracting setting with intermediation costs. These costs increase with the individual and aggregate level of debt, which leads to a spread between lending and deposit rate. If the debt overhang is too large, that is, the interest rate spread is large, deleveraging of borrowers may lead to a negative natural rate of interest.

In summary, financial frictions invoke important dynamic feedback mechanisms between debt and economic activity. Moreover, models embedded in a New Keynesian framework mostly build on the interest rate channel of monetary policy. Several other contributions, however, highlight the importance of the health of financial intermediaries for fluctuations in prices and debt. To gain a broader basis for discussion in later sections, it is therefore reasonable to choose one model of each strand.
IV MODEL OF DELEVERAGEING

Eggertsson and Krugman (2012) (EK) and Benigno, Eggertsson and Romei (2013) (BER) provide a New Keynesian model with heterogeneous agents and dynamic deleveraging. The basic story can be summarised in the following way: there are borrowers and savers in the economy. In periods of high optimism, borrowers acquire debt to finance consumption expenditures, thus the economy as a whole builds up leverage. When people realise that the newly issued debt might in fact not be sustainable, the economy enters a period of low optimism. All of a sudden borrowers realise they are in fact overleveraged and have to pay down their debt - a Minsky moment. As borrowers enter a process of fast deleveraging, someone has to compensate for the decline in expenditure. By cutting the policy rate and driving down the real interest rate, the central bank aims to induce savers to increase their consumption expenditures in order to preserve full employment. However, if the debt-overhang is too large, this might lead to a scenario where the nominal interest rate needed to induce sufficient spending of savers may be negative. That is, negative real interest rates might make the zero bound on the nominal interest rate binding. As the central bank cannot reach its target interest rate, the economy enters a recession.

The model presented in this section is essentially a blend of EK and BER. As can be seen below, the model may serve as a generalisation of the standard New-Keynesian textbook model.

A. Households

The economy is populated by two types of households on an interval \([0, 1]\), borrowers with measure \(\chi_b = \chi\) and savers with measure \(\chi_s = 1 - \chi\). Denote the group of savers by \(s\) and borrowers by \(b\), and the group index by \(c \in \{s, b\}\). Savers are endowed with a discount factor \(\beta_s \in (0, 1)\), while borrowers’ discount factor \(\beta_b\) satisfies \(0 < \beta_b < \beta_s < 1\). Each individual \(i \in [0, 1]\) solves

\[
\max \mathbb{E}_t \sum_{T=t}^{\infty} \beta_c^{T-t} (u(C^c_T) - v(h^c_T))
\]

subject to the budget constraint

\[
B^c_{t-1} + P_tC^c_t + T^c_t = \frac{B^c_t}{1 + \delta_t} + W_t h^c_t + Z_t^c + I^c_t,
\]

Ek provide an extension building on Cúrdia and Woodford (2009), in which they embed the deleveraging mechanism developed in a two-period set-up in a DSGE framework in the appendix to their paper.
where a positive $B_i^t$ is debt of individual $i$, $C_i^t$ the consumption bundle of individual $i$ and $h_i^t$ labour supply of individual $i$. $W_i$ denotes the nominal wage rate and $P_t$ the aggregate price level; $Z_i^t$ are profits from operating retail firms, while $I_i^t$ are profits from financial intermediation; $T_i^t$ are lump-sum taxes.

Borrowing and lending between agents is only possible via financial intermediaries. Savers receive the risk-free short term interest rate $i_t$, while borrowers face the borrowing rate

$$1 + i_i^t = (1 + i_t)\tilde{\omega}\left(\frac{b_i^t}{d_t}, \frac{b_t}{d_t}\right),$$

which is proportional to the nominal interest rate $i_t$ and specific to borrower $i$, where $\tilde{\omega}(\cdot, \cdot) > 1$ is a borrowing premium depending on individual $i$’s real debt $b_i^t = B_i^t / P_t$ and the aggregate level of debt $b_t = \left(\int \chi b_i^t di\right) / \chi$. If the individual level of debt exceeds an exogenously given risk-free level of real debt $d_t$, the borrowing premium increases, i.e. $\tilde{\omega}_b(\cdot, \cdot) \geq 0$. Similarly, the borrowing premium is assumed to be increasing with respect to the aggregate level of debt, i.e. $\tilde{\omega}_b(\cdot, \cdot) > 0$. Note that even if both individual and aggregate real debt are equal to $d_t$, then there is a spread $\tilde{\omega} \equiv \tilde{\omega}(1, 1) > 1$.

Given the positive spread between borrowing and deposit rates, aggregate profits from intermediation are given by

$$I_t = \int \chi \left(\frac{1}{1 + i_t} - \frac{1}{1 + i_i^t}\right) b_i^t di > 0,$$

where the profits are assumed to be distributed to savers only.\(^{11}\)

Given the risk-free rate $i_t$, savers’ optimisation problem implies the following consumption Euler equation for all individuals of the group of savers $s$:

$$1 = \beta_s E_t \frac{1 + i_t}{\Pi_{t+1}} \frac{u_c(C_{t+1}^s)}{u_c(C_t^s)},$$

where $\Pi_t = P_t / P_{t-1}$. As borrowers’ debt decisions depend on their individual borrowing costs, they are not price-takers. The consumption Euler equation for each individual $i$ in the group of borrowers depends on the elasticity of the borrowing premium with respect to individual real debt. However, in equilibrium, borrowers choose the same level of debt $b_i^t = b_t$,\(^{10}\)

---

\(^{10}\)The positive spread between the borrowing and deposit rate arises due to different discount factors. This ensures that $\omega(\cdot, \cdot)$ is continuously differentiable in a neighbourhood around the steady state. Unfortunately, this property is overlooked in BER. Moreover, the steady state interest rate spread is uniquely determined by the discount factors.

\(^{11}\)It seems reasonable to assume that indebted agents are excluded from receiving the profits of intermediation. Both EK and BER note that the qualitative results do not depend on this assumption.
which implies that the consumption Euler equation for borrowing individuals is

\[ 1 = \beta b E_t \frac{1 + i_t^b}{\Pi_{t+1}} \frac{u_c(C_{t+1}^b)}{u_c(C_t^b)}, \]

(2)

where \( \epsilon(\cdot) \equiv (b_t/d_t)(\omega(b_t)/\omega(\cdot)) \) is the elasticity of the borrowing premium with respect to real debt, and \( \omega(b_t/d_t) \equiv \bar{\omega}(b_t/d_t, b_t/d_t) \). Note that the equilibrium borrowing rate is now

\[ 1 + i_t^i = 1 + i_t^b = (1 + i_t)\omega b_t/d_t \]

(3)

for each borrower \( i \).

Optimal labour supply of each saver is given by

\[ \frac{v_h(h_s^s)}{u_c(C_s^s)} = \frac{W_t}{P_t}, \]

(4)

and similarly for each borrower by

\[ \frac{v_h(h_b^b)}{u_c(C_b^b)} = \frac{W_t}{P_t}. \]

(5)

**B. Firms**

There is a continuum of firms \( j \in [0,1] \), each of which produces a differentiated intermediate good \( y_t(j) \), but all use an identical technology represented by the following constant returns to scale production function\(^\text{12}\)

\[ y_t(j) = h_t(j). \]

Perfectly competitive firms assemble the intermediate goods with a Dixit and Stiglitz (1977) technology

\[ Y_t = \left( \int_0^1 y_t(j)^{-\frac{\theta}{\theta-1}} dj \right)^{\frac{\theta}{\theta-1}}, \]

where \( \theta \) is the elasticity of substitution between goods. Final good firms maximise \( P_tY_t - \int_0^1 p_t(j)y_t(j) dj \) subject to the production function, which yields the demand functions for the intermediate goods

\[ y_t(j) = \left( \frac{p_t(j)}{P_t} \right)^{-\theta} Y_t \quad \text{with} \quad P_t \equiv \left( \int_0^1 p_t(j)^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \]

\(^{12}\)By keeping the model as simple as possible, it is convenient to abstract from any changes in technology, so \( A_t \equiv \bar{A} \) is normalised to 1. In addition, the constant returns to scale assumption yields some nice properties which are exploited in appendix A.
The profit maximisation problem of each intermediate good firm in period \( t \) is hence given by

\[
 z_t(j) = (1 + \tau)p_t(j)y_t(j) - W_T h_T(j),
\]

where \( \tau \) is an employment subsidy financed by means of lump-sum taxes levied on individuals according to

\[
 T_t^i = \tau P_t Y_t.
\]

Furthermore, pricing à la Calvo (1983) is assumed. That is, firms only get to revise their prices with probability \( (1 - \alpha) \) at the beginning of each period, while the remaining firms leave their price unchanged. Furthermore, firms take the exogenously set inflation target \( \Pi \) into account. Each firm \( j \) that revises its price at time \( t \) chooses \( p_t^* \) to maximise its discounted profits

\[
 \max_{p_t^*} \mathbb{E}_t \left\{ \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T \left[ (1 + \tau)\Pi^{T-t} \frac{p_t^*}{P_T} y_T(j) - \frac{W_T}{P_T} h_T(j) \right] \right\},
\]

where \( \lambda_t \) is a weighted average of the marginal utilities of savers and borrowers since profits are risk-shared across the two classes, and \( \beta = (1 - \chi) \beta_s + \chi \beta_b \). The first order condition of this problem implies an optimal price \( p_t^*/P_t \). Together with

\[
 \frac{p_t^*}{P_t} = \left( 1 - \alpha \left( \frac{\Pi}{\Pi_T} \right)^{\theta-1} \right)^{1/\theta},
\]

which also follows from Calvo-pricing.

The maximisation problem given by (*) under Calvo-pricing, together with the Dixit-Stiglitz aggregator, implies the New Keynesian aggregate supply equation given by

\[
 \left( 1 - \alpha \left( \frac{\Pi}{\Pi_T} \right)^{\theta-1} \right)^{1/\theta} = \mu \frac{E_t}{F_t} \tag{6}
\]

with \( \mu = \theta/[(\theta - 1)(1 + \tau)] \) and auxiliary variables \( E_t \) and \( F_t \) recursively defined as

\[
 E_t = \lambda_t Y_t \frac{W_t}{P_t} + \alpha \beta E_t \left( \frac{\Pi_t+1}{\Pi} \right)^\theta E_{t+1} \tag{7} \text{ and}
\]

\[
 F_t = \lambda_t Y_t + \alpha \beta E_t \left( \frac{\Pi_t+1}{\Pi} \right)^{\theta-1} F_{t+1}. \tag{8}
\]

The derivation of this result can be found in appendix A.
Define price dispersion due to sticky prices as \( \Delta_t = \int_0^1 \left( \frac{p_t(j)}{P_t} \right)^{-\theta} di \) with the following law of motion\(^\text{13}\)

\[
\Delta_t = (1 - \alpha) \left( 1 - \alpha \frac{(\Pi_t \Pi_t)^{\theta-1}}{1 - \alpha} \right)^{\frac{\theta}{\theta-1}} + \alpha \Delta_{t-1} \left( \frac{\Pi_t}{\Pi_t} \right)^{\theta}. \tag{9}
\]

### C. Equilibrium

Realising that aggregate labour supply is \( h_t = (1 - \chi)h^s_t + \chi h^b_t \) and recalling the constant returns to scale assumption together with the individual demand functions, yields

\[
(1 - \chi)h^s_t + \chi h^b_t = \int_0^1 y_t(j) dj = \int_0^1 Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta} dj = Y_t \Delta_t, \tag{10}
\]

where the aggregate resource constraint is simply given by

\[
Y_t = (1 - \chi)C^s_t + \chi C^b_t. \tag{11}
\]

Finally, the borrowers’ budget constraint gives the evolution of aggregate real debt. Substituting \( Z^i_t = (1 + \tau)P_t Y_t - W_t h_t, T^i_t = \tau P_t Y_t, \) noting the composition of aggregate labour supply, and recalling that \( I^i_t = 0 \) for borrowers, yields

\[
\frac{b_t}{1 + i^b_t} = \frac{b_{t-1}}{1 + i^b_{t-1}} + C^b_t - \frac{W_t}{P_t} (1 - \chi)(h^b_t - h^s_t). \tag{12}
\]

To close the model, monetary policy is assumed to follow a general form

\[
i_t = \max(0, \phi X_t), \tag{13}
\]

where \( \phi \) is a vector of exogenous policy parameters and \( X_t \) is a vector of endogenous variables. It suffices to provide a specific policy rule for the linearised model. Now an equilibrium for the model can be properly defined.

**Definition 1.** An equilibrium is a set of endogenous stochastic processes \( \{Y_t, C^s_t, C^b_t, h_t, W_t, E_t, F_t, \Pi_t, \Delta_t, i^b_t, i_t, b_t, \lambda_t, b_t\}_{t=t_0}^{\infty}, \) exogenous process \( \{d_t\}_{t=t_0}^{\infty}, \) and given initial conditions \( d_{t_0-1} \) and \( \Delta_{t_0-1} \), such that equilibrium conditions (1)-(13) are fulfilled. In other words, the endogenous variables are consistent with optimal inter-temporal decisions by individuals and firms, given their objective functions and constraints, such that all markets clear at all times.

Before the effects of a deleveraging shock can be discussed, the steady states for the linearisation have to be pinned down.

\(^\text{13}\)For details on the derivation see appendix B.
D. Steady State

As a starting point, consider an initial steady state where the economy is in a highly leveraged state with $d_t = d^b$. In steady state, the inflation rate is equal to the inflation target $\Pi_t = \Pi$. This implies that there is no price dispersion in steady state, i.e. $\Delta_t = 1$. Given the consumption Euler equations, savers earn a risk-free rate of

$$1 + i = \beta_s^{-1} \Pi,$$

while borrowing households pay a risk-free rate

$$1 + i_b = \beta_b^{-1} \Pi.$$  

The relationship between the two interest rates pins down the steady-state spread $\bar{\omega} = \beta_s \beta_b^{-1}.14$ Note that this implies positive profits from intermediation for savers in steady state. This means that savers’ steady-state income from intermediation falls in response to a drop in $d$.

In order to simplify notation, the production subsidy $\tau$ is assumed to eliminate the static distortion due to monopolistic competition, i.e. $\mu = \frac{\theta}{(1 - \theta)(1 + \tau)} = 1$, and the steady-state real wage is set as $W/P = 1/\mu = 1$. Furthermore, it is convenient to normalise steady-state aggregate output to $Y = 1$, which implies $h = 1.15$

Similar to EK, the steady-state values for $C_b$, $C_s$, $h_b$ and $h_s$ can be backed out from the optimal labour supply, the evolution of aggregate debt and the aggregate resource constraint.

E. Linearisation

This section presents the linear approximation of the equilibrium conditions around the steady state defined above. Moreover, it can be shown that the linearised version of the model represents a more general version of the standard New Keynesian model by incorporating heterogeneous agents and financial frictions.16 Denote the deviations of endogenous variables from their steady-sates as by $\hat{y}_t \equiv (Y_t - Y)/Y$, $\hat{c}^c_t \equiv (C^c_t - Y)/Y$ and $\hat{h}^c_t \equiv (h^c_t - h^c)/h^c$ for each $c \in \{s, b\}$, $\hat{b}_t \equiv (b_t - d^b)/d^b$, $\hat{d}_t \equiv (d_t - d^b)/d^b$, $\pi_t \equiv \ln \Pi_t$, $\hat{i}_t \equiv \ln(1 + i_t)/(1 + i)$ and $\hat{i}^b_t \equiv \ln(1 + i^b_t)/(1 + i^b)$.

14Note that the consumption Euler equation can be rewritten as $\beta_s^{-1} \Pi = (1 + i^b)(\bar{\omega} - \tilde{\omega}_s(1, 1))/\bar{\omega}$ which implies that $\tilde{\omega}_s(1, 1) = 0$. That is, marginal cost of additional individual borrowing is zero in steady state.

15Note that this normalisation is justified by the assumption that $\tau$ eliminates the effect of price stickiness on steady-state output, which makes the steady-state values of output in the flexible price and the sticky price setting equivalent.

16Examples of similar models but with a different focus are Cúrdia and Woodford (2009, 2010, 2011).
The consumption Euler equations (1) and (2) can then be rewritten as

\begin{align}
E_t \hat{c}^c_{t+1} - \hat{c}^c_t &= \sigma \left( \hat{h}_t - E_t(\pi_{t+1} - \pi) \right) \\
E_t \hat{c}^b_{t+1} - \hat{c}^b_t &= \sigma \left[ \hat{b}^b_t + \lambda \left( \hat{b}_t - \hat{d}_t \right) - E_t(\pi_{t+1} - \pi) \right]
\end{align}

(16) (17)

where \( \pi \equiv \ln \Pi, \sigma \equiv -u_{cc}Y/u_c \) and \( \lambda \equiv \epsilon_b(1) > 0 \). Linearising (3) gives an approximation for the spread between borrowing and deposit rate

\[
\hat{\delta}_i^b = \hat{i}_t + \phi \left( \hat{b}_t - \hat{d}_t \right),
\]

(18)

where \( \phi \equiv \omega_b(1) > 0 \) is the steady-state elasticity of the borrowing premium with respect to real debt. Linearisation of the evolution of real debt (12) yields

\[
\frac{\bar{b}}{1 + \bar{b}} \left( \hat{b}_t - \hat{\delta}_i^b \right) = \frac{\bar{b}}{\Pi} \left( \hat{b}_{t-1} - (\pi_t - \pi) \right) + \hat{\delta}_i^c - \hat{\delta}_i^b - \hat{h}_t^b + \hat{h}_t^c,
\]

(19)

where \( \bar{b} \equiv b/Y \).

The aggregate resource constraint (11) gives

\[
\hat{y}_t = (1 - \chi)\hat{c}^c_t + \chi \hat{c}^b_t
\]

(20)

and, noting that price dispersion drops on a first-order approximation,

\[
\hat{y}_t = (1 - \chi)\hat{h}_t^c + \chi \hat{h}_t^b.
\]

(21)

Aggregate labour supply can be summarised as

\[
\varphi \hat{h}_t^b + \sigma^{-1} \hat{c}_t^b = \varphi \hat{h}_t^c + \sigma^{-1} \hat{c}_t^c
\]

(22)

where \( \varphi \equiv \nu_{hb}Y/\nu_h \). Equations (16)-(22) describe the aggregate demand block of the economy.

Log-linear approximations of equations (6)-(8) yield the New Keynesian Phillips curve that represents the aggregate supply block of the model

\[
\pi_t - \pi = \kappa \hat{y}_t + \beta E_t(\pi_{t+1} - \pi),
\]

(23)

where \( \kappa \equiv (1 - \alpha)(1 - \alpha \beta) / (\sigma + \varphi) / \alpha \).

Finally, monetary policy is assumed to follow a standard textbook specification taking into account the ZLB

\[
\hat{i}_t = \max(\beta^{-1} \Pi - 1, \phi_p \pi_t + \phi_y \hat{y}_t).
\]

(24)

\[\text{Note that } \lambda > 0 \text{ is guaranteed by assuming that the cross derivative } \omega_{bb}(1,1) > 0 \text{ since } \epsilon(1) = \omega_{b1}(1,1)/\omega(1,1).\]

\[\text{To ensure } \phi > 0, \text{ the assumption made above that } \tilde{\omega}_b(\cdot,\cdot) > 0 \text{ suffices since } \omega_b(1) = (\tilde{\omega}_b(1,1) + \hat{\omega}(b,1))/\hat{\omega}(1,1) \text{ where } \omega_b(1,1) = 0.\]
Equations (16)-(23), together with the exogenous process \( \{d_t\}_{t=t_0}^{\infty} \), monetary policy rule (24) and the initial condition \( \hat{b}_{t_0-1} \), determine the equilibrium processes of the endogenous variables \( \{\hat{y}_t, \hat{c}_{bt}^s, \hat{c}_{cs}^s, \hat{h}_{bt}^s, \hat{h}_{cs}^s, \pi_t, \hat{i}_t, \hat{i}_{bt}, \hat{b}_t\}_{t=t_0}^{\infty} \).

At this point it has to be emphasised that the linearised version of the heterogeneous agent model derived above is closely linked to the standard textbook New Keynesian model, e.g. Woodford (2003) or Galí (2008). Combining equations (16)-(18) and (20), yields
\[
\hat{y}_t = E_t \hat{y}_{t+1} - \sigma \left[ i_t - E_t (\pi_{t+1} - \pi) - \hat{r}_n^s \right],
\]
where the natural rate of interest \( \hat{r}_n^s \) is now endogenous and given by
\[
\hat{r}_n^s = -\chi (\lambda + \phi) (\hat{b}_t - \hat{d}_t).
\]

Moreover, the aggregate supply schedule from above is exactly the same as in the standard model. The standard New Keynesian model with an exogenous natural rate of interest is a nested special case of the model outlined above. To determine the natural rate of interest in the above setting, one has to figure out the evolution of debt first. If the real value of debt is higher than the safe value of debt, borrowers find themselves in a situation where private debt is too high. This is equivalent to a negative shock to the natural interest rate, which in turn implies a negative shock to the demand schedule. In particular, if the negative shock to \( \hat{d}_t \) is large enough, one can easily get a negative natural rate of interest. Conversely, if real debt is below the safe level, borrowers will have an incentive to lever up and increase their spending, which is equivalent to a positive demand shock.

F. Simulation

To conclude the characterisation of the model, consider now simulations of the linearised model (a) without the ZLB and (b) with the ZLB binding. The model is implemented in Dynare (cf. Adjemian et al., 2011) and the model file to replicate the figures can be found in appendix C. To solve the model for a binding ZLB, Guerrieri and Iacoviello (2014) provide a convenient algorithm which is used for solving the linear model within the Dynare environment, see appendix D. In a nutshell, the algorithm is a piecewise linear perturbation method that checks in every step whether the ZLB is binding or not.

Table 1: Baseline parameter set for the linearised deleveraging model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_s )</td>
<td>0.995</td>
</tr>
<tr>
<td>( \beta_b )</td>
<td>0.99</td>
</tr>
<tr>
<td>( \sigma )</td>
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</tr>
<tr>
<td>( \varphi )</td>
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</tr>
<tr>
<td>( \chi )</td>
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</tr>
<tr>
<td>( \phi )</td>
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<tr>
<td>( \lambda )</td>
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<tr>
<td>( \kappa )</td>
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<tr>
<td>( \bar{b} )</td>
<td>10</td>
</tr>
<tr>
<td>( \sigma_d )</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Table 1 shows the parameters used in all simulations. Using the common calibration of \( \beta_s \) ensures that the annual risk-free nominal interest rate is
about 2 percent.\footnote{Note that the steady-state annual nominal interest rate is \( i^{\text{ann}} = 4(\beta^{-1} - 1) \).} Borrowers’ discount factor \( \beta_b \) is just slightly below those of savers. This relationship is crucial as the classification of indebted and saving agents emerges endogenously from some agents being less patient than others. As in Gali (2008), it is assumed that \( \sigma = 1 \) (implying log Frisch elasticity of labour supply). In line with Iacoviello (2005) and BER, the share of borrowers in the economy is assumed to be \( \chi = 0.4 \). BER calibrate \( \bar{b} \) to match an initial ratio of aggregate debt to GDP, denoted by \( d^{\text{gdp}} \), of 100 percent. That is, \( b^{\text{gdp}} = (\chi b)/(4y) = 1 \) which implies \( b = b/y = 4/\chi \).\footnote{EK, on the other hand, consider an initial value of aggregate debt to GDP of 130 percent, while both BER and EK contributions analyse the effects of a 30 percent drop in \( d \). Note that (12) depends on \( b \) which means that the deleveraging process has a level effect. That is, if aggregate debt is very high, then the effect of the deleveraging process is even more pronounced, as discussed further below.} The simulation analyses the transition between two steady states as the safe level of debt drops unexpectedly from \( d_h \) to \( d_l \), with \( d_l < d_h \). To stick to the above notation, the following process for \( \hat{d}_t \) is assumed

\[
\hat{d}_{t+1} = \hat{d}_t - \varepsilon_{t+1}. 
\]

In order to get a drop in \( d \) of 30 percent, one simply has to assume that \( \varepsilon_t \overset{i.i.d.}{\sim} (0, \sigma_d^2) \), where \( \sigma_d \equiv (1 - d_h/d_l) \). An unanticipated one-time shock to \( d \) at period 1 with standard deviation \( \sigma_d \) creates a Minsky-moment, in which borrowers suddenly find themselves at too high debt levels. This triggers the deleveraging dynamics that will be discussed shortly.

As in Woodford (2003), the measure of the degree of price flexibility \( \kappa \) is assumed to be 0.02.\footnote{Note that \( \kappa \) is a function of several other structural parameters. However, for the linearised model, it suffices to just calibrate \( \kappa \).} The calibrations of the steady-state elasticity of the borrowing premium \( \phi \) and the steady-state derivative of the elasticity of the premium seem somewhat arbitrary, as they lack empirical backing at the moment. However, both parameters essentially determine the impact of the drop in \( d \) via equations (17) and (18). Pragmatically, values are chosen in order to generate meaningful results.

**Scenario (a).** Figures 2 and 3 show the effects of the deleveraging shock without the ZLB. The central banking is assumed to target a 2 percent annual inflation rate. There are two cases in this scenario: one where the central bank is able to perfectly target 2 percent inflation, and one where the central bank follows standard textbook parametrisation.

In both cases, the negative shock to the risk-free level of debt triggers an increase in the spread between the borrowing and deposit rates, since borrowers suddenly find themselves far away from the risk-free borrowing limit. As the cost of borrowing is high, borrowers start paying down their debt by
both decreasing their consumption and increasing their total hours worked, while savers do exactly the opposite. If prices are perfectly flexible (which, in this case, is equivalent to successful inflation targeting), household consumption smoothing completely offsets the effect on aggregate output. On the other hand, if price frictions do matter (i.e. under a standard parametrisation of the policy function), output and inflation are affected by the deleveraging process. As the policy rule does not trigger a response of the interest rate which is large enough to offset the decline in consumption of borrowers, output and hence prices are negatively affected.

Note that as the natural rate of interest becomes negative, the risk-free rate of savers declines. This means that consumption in the current period is relatively less expensive now than it was before. This is why savers increase their consumption while borrowers deleverage. As there is no bound on the nominal interest rate, the real interest rate can adjust such that savers can compensate for the drop in consumption of borrowers. However, the necessary decline in the interest rate is quite large which poses problems once the ZLB is taken into account.

It has to be emphasised that once the central bank cannot successfully target inflation, the deleveraging process has a negative effect on output and inflation, even though the ZLB is not binding. This means that shifts in the risk-free level of debt might trigger considerable fluctuations in output.

**Scenario (b).** The key mechanism behind the aforementioned deleveraging process is that the risk-free interest rate declines in response to the lower consumption of borrowers inducing savers to increase their, thereby compensating for the loss of borrower consumption. Given an inflation target of 2 percent, the real interest rate of savers can be at least -2 percent when the nominal rate is zero.

Figures 4 and 5 give the simulation results when the ZLB is binding and the central bank tries to perfectly target its inflation goal. The economy experiences a significant drop both in output and in the rate of inflation as the central bank cannot set its interest rate at the desired level. Lower output means lower income for borrowers who are now forced to decrease consumption even further in order to reduce their debt levels. This slows down the deleveraging process and the borrowing rate increases. Due to the endogeneity of the natural interest rate, a slower deleveraging process caused by the ZLB implies a slower recovery of the natural rate of interest. Consequently, the ZLB on the nominal interest rate is binding for a longer time period than if it had been caused by a purely exogenous shock.

Quantitatively, the deleveraging shock can be quite large, particularly

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22Note that households can only smooth consumption by varying their labour supply. For instance, a modified version of the model including capital would give households a further margin for consumption smoothing. However, this aspect goes beyond the scope of this thesis and is postponed to future research.
when the economy is close to the ZLB. BER emphasise that the effect on output with an endogenous natural rate of interest, which incorporates a feedback loop with output and the policy rate, is much more pronounced than with an exogenous natural rate of interest. Put differently, the output gap lowers the natural rate of interest even further, thereby making the nominal interest rate binding for even longer. As aggregate demand also depends on expected output, expected inflation and expected future nominal interest rates, all these factors will depress demand today. Thus both the persistence of the crisis, as well as its impact are higher with an endogenous natural rate than with an exogenous one. Furthermore, BER argue that the implications for optimal monetary policy are similar to Eggertsson and Woodford (2003), who show that keeping the nominal interest rate low for

**Figure 2:** Impulse response functions following a 30 percent deleveraging shock without the ZLB.
a period of time longer than really needed for inflation targeting can lead to a small output boom and inflation in the very short run. However, due to the endogeneity of the natural rate of interest, the commitment has to be even stronger in the scenario simulated above.

EK, on the other hand, strongly argue in favour of fiscal policy in countering the negative effects of the binding ZLB. When the economy is at the ZLB, their results suggest that the government spending multiplier should be greater than one, since monetary policy is not able to offset the effect of fiscal policy. As long as borrowers benefit from an increase in government spending, the deleveraging process will take less time and substantially reduce the length of the crisis period. BER further note that the fiscal multiplier is larger than in the standard New Keynesian model with ZLB,
since government spending also affects the natural rate of interest through the deleveraging process. The size of the multiplier of course depends on the distribution of taxes and spending between savers and borrowers. If savers finance the increase in government spending, which mostly targets overleveraged borrowers (essentially a redistribution from savers to borrowers), the multiplier becomes relatively large. Conversely, if only borrowers face the tax burden and transfers are shared across all individuals, the impact of government spending can also be negative. These results are fairly straight-forward as the length and intensity of the deleveraging process of borrowers essentially determines the decline in the natural rate of interest and, consequently, output and inflation.

Figure 4: Impulse response functions following a 30 percent deleveraging shock with and without the ZLB under perfect inflation targeting.
Interestingly, the magnitude of the effects is not scale-invariant (see footnote 20). While the drop in the natural rate of interest only depends on the relative change in the risk-free debt level (i.e. the debt overhang), the impact on output and inflation depends also on the level of aggregate debt, as depicted in figure 6. This means that the more indebted the economy, the worse the crisis following a deleveraging shock. This stems from the fact that the steady-state value of consumption of borrowers ultimately depends on the safe level of debt $d$ before the shock. The higher $d$, the higher the steady-state output share of borrowers’ consumption. If highly indebted agents cut down their consumption expenditures, falling prices make it even harder for them to decrease their debt levels. If the initial debt level is high enough, deflation might kick in despite monetary policy targeting their
positive inflation goal. This leads to debt-deflation and amplifies the initial drop in borrowing. Hence, the deleveraging of borrowers can have stronger effects when they are highly indebted. This is consistent with Minsky’s view that calm periods of economic volatility, which may lead to high levels of private debt, can be followed by severe financial crises.

Figure 6: Impulse response functions following a 30 percent deleveraging shock at different debt levels (with ZLB and perfect inflation targeting).
V Model of Money Creation

Brunnermeier and Sannikov (2014a, b) provide a new modelling approach by incorporating a financial sector in macroeconomic models to tackle issues centring around financial stability and financial frictions. Their economy is populated by three different groups of agents: borrowers, savers and intermediaries. Borrowers fulfil one or more of the following characteristics: they are entrepreneurs and more productive than other agents, or more impatient, risk-tolerant and optimistic than others. Most importantly, they want to hold more capital than their own resources would allow. In these models, there are financial frictions in the sense that direct contracts are incomplete, e.g. savers cannot monitor borrowers effectively, and financial intermediaries that can partially mitigate these frictions. Similar to Holmstrom and Tirole (1997), financial intermediaries take over at least some of the risk of borrowers. How much risk the financial sector can take on, however, depends on its overall health, or more specifically, its net worth. The state of the economy can therefore be described by the aggregate net worth of the production and the intermediation sectors. Exogenous macroeconomic shocks essentially affect the return on capital, whereas the price of capital also depends on the net worths of borrowers and intermediaries.

In addition to risky capital, Brunnermeier and Sannikov (2014b) (BS) introduce outside money as a safe store of value. Intermediaries create inside money by issuing debt-obligations. The model is able to derive endogenous systemic risk dynamics manifested in deflationary and liquidity spirals, thereby incorporating elements in the spirit of Fisher and Minsky. This section draws on the model of BS. More specifically, this section replicates the baseline model of the original paper and includes a simple interest rate rule rule of the monetary authority. The original paper includes conventional and unconventional monetary policy at the same time.

A. Agents

The model economy consists of households and intermediaries. The optimisation problem of all agents is twofold: they choose consumption rates to maximise utility subject to their wealth levels and they allocate their port-

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23More specifically, this section replicates the baseline model of the original paper and includes a simple interest rate rule rule of the monetary authority. The original paper includes conventional and unconventional monetary policy at the same time.
folio between different assets in order to maximise returns. That is, the income of the individuals is generated by capital gains on investments in different assets. All households are assumed to have logarithmic utility of the form

$$\mathbb{E} \left[ \int_0^\infty e^{-rt} \log c_t \, dt \right],$$

where $c_t$ is the rate of consumption and $r \in (0, 1)$ is the discount rate of households. Some households are savers and some are borrowers. The latter are assumed to value physical capital higher since they have direct access to the production technology. For simplicity, the total wealth of borrowers is assumed to be zero, i.e. savers own all household wealth. Without intermediaries, borrowers have to borrow physical capital from savers in order to produce output. With an intermediary sector, savers can deposit some of their funds with the intermediaries who can diversify borrowers’ risk and thus provide an additional funding channel to finance borrowers’ projects.

Physical capital $k_t$ used by borrowers produces output at rate $(a - \iota_t)k_t$, where $a > 0$ is a technology parameter and $\iota_t$ is the rate of investment. New capital can be built through investment and evolves according to

$$\dot{k}_t = (\Phi(\iota_t) - \delta)k_t,$$

where $\Phi$ represents a standard investment technology with adjustment cost, i.e. $\Phi(0) = 0$, $\Phi'(0) = 1$, $\Phi''(\cdot) > 0$ and $\Phi''(\cdot) \leq 0$. That is, for $\iota_t \geq 0$ there are decreasing returns to scale and for $\iota_t < 0$ there is technological illiquidity, as adjustment costs imply irreversibility of investment.\(^24\) In the absence of investment, physical capital depreciates at rate $\delta$. Taking capital holdings of savers into account, total capital in the economy $K_t$ evolves according to

$$\dot{K}_t = (\Phi(\iota_t) - \delta)K_t.$$

Intermediaries also have logarithmic utility of the form

$$\mathbb{E} \left[ \int_0^\infty e^{-\rho t} \log c_t \, dt \right],$$

where $c_t$ is their rate of consumption and $r < \rho < 1$ is the discount rate. In other words, intermediaries are more impatient than households. In addition, intermediaries have certain advantages with respect to macroeconomic shocks.

Macroeconomic shocks follow a Poisson process with intensity $\lambda$. In the case of a shock, a fraction $\phi$ of borrowers steal physical capital and become savers. Analogously, savers lose their entire capital that they rented out with probability $\phi$ and become borrowers. This can be interpreted as some agents

\(^{24}\)Irreversible investment means that capital cannot be converted back to consumption goods. Moreover, the depreciation rate can here be interpreted as the average duration of the capital goods (Brunnermeier and Sannikov, 2012).
being subjected to failing investments, hence losing their invested capital to other agents. Intermediaries are able to reduce the probability of losing capital to $\phi < \bar{\phi}$ since they can diversify across borrowers and are assumed to have a superior monitoring technology. In the event of a macroeconomic shock, however, intermediaries lose a fraction $\phi$ of their wealth. That is, shocks affect the net worth of intermediaries, but leave the total quantity of capital $K_t$ unaffected.

### B. Assets and Returns

There are two types of assets across which agents can allocate their wealth: money and capital. All asset values are measured in units of output, i.e. the price of output is one. Let $q_t$ be the equilibrium price of one unit of physical capital, while $p_t$ is the price of money per unit of physical capital.\(^{25}\) The total amount of physical capital depends on aggregate investment and the allocation of capital between borrowers and savers. Money is assumed to be backed by taxes and pays a floating interest rate $i_t \geq 0$ set by the monetary authority. Fiscal policy taxes output at rate $\tau \in [0, 1)$ and buys money at the current price, while the monetary authority prints money and takes outside money as deposits to finance interest payments.\(^{26}\) Both the prices of money and the price of capital are endogenously determined. The total amount of real wealth in the economy is therefore equal to $(p_t + q_t)K_t$.

Savers choose to hold their wealth as money or capital. They can either hold inside money, by making a deposit with an intermediary, or hold outside money directly. As each saver loses her entire capital with intensity $\lambda \phi$, holding physical capital only is very risky. The expected utility of holding capital only is $-\infty$ which ensures that there is a demand for money.

Intermediaries create inside money by providing funds to invest in capital projects and accepting deposits from savers. They have to guarantee to repay the savers even in the event of a macro shock. As deposits are denominated in money, inside money created by the intermediaries can be viewed as an imperfect substitute for outside money. However, the amount of inside money intermediaries are willing to create depends on their risk-taking capacity which is essentially determined by their net worth.

Furthermore, intermediaries are assumed to issue debt only and no equity, i.e. they have to entirely absorb all losses from their lending activity. This

\(^{25}\)For this definition of the price of money to make sense, the total money supply has to be normalised to one and remain unchanged. Let $P_t \equiv P$ be the value of the total money stock (in terms of output). Then a change in $p_t = P_t/K_t \equiv P/K_t$ implies a change in the value of the money supply. Note that a growing capital stock implies that the price of money is decreasing. The main advantage of this assumption, however, is that it keeps everything scale invariant with respect to the size of the economy.

\(^{26}\)Note that $i_t$ is the single policy instrument in this setting, while BS also include long-term bonds issued by the central bank. The implications of this additional policy instrument are discussed below.
assumption creates the necessary financial friction which comes into play below.

Before the equilibrium can be characterised formally, one has to look at the returns households and intermediaries receive from money and capital. Denote by

\[ \mu_p^t = \frac{\dot{p}_t}{p_t} \quad \text{and} \quad \mu_q^t = \frac{\dot{q}_t}{q_t} \] (25)

the endogenous equilibrium laws of motion of prices without shocks and by \( \hat{p}_t \) and \( \hat{q}_t \) the price levels in the event that a shock hits at time \( t \).

First, consider an investment in physical capital of \( q_t k_t \). It earns a return of

\[
r_t^K = \left(1 - \tau\right)\left(a - \iota_t\right) q_t + \mu_q^t + \Phi(\iota_t) - \delta.
\] (26)

in the absence of shocks. After a shock, the value of capital owned by households drops to zero with probability \( \phi \) and changes by a factor of \( \hat{q}_t/q_t \) with probability \( 1 - \phi \). However, capital held by an intermediary changes by \( (1 - \phi)\hat{q}_t/q_t \).

Moreover, the optimal equilibrium rate of investment solves the first-order condition

\[
\max_{\iota_t} \Phi(\iota_t) - \frac{\iota_t}{q_t} \Rightarrow \Phi'(\iota_t) = \frac{1}{q_t}.
\] (27)

As fiscal policy buys money at the current market price, the flow of payments to money holders is \( \tau a K_t \). In addition, monetary policy pays the floating interest rate \( i_t \). Since the total value of money in the economy is \( p_t K_t \), the return on money is

\[
r_t^M = \frac{\tau(a - i_t)}{q_t} + i_t + \mu_p^t + \Phi(\iota_t) - \delta,
\] (28)

where the value of money changes by a factor \( \hat{p}_t/p_t \) following a shock.

C. Optimal Portfolio Choice

Logarithmic utility has the advantage that portfolio and consumption decisions are independent of each other and consumption decisions only depend on consumers’ wealth levels. Put differently, consumption is independent of investment opportunities. Moreover, in case of infinitely-lived agents, the

\(^27\) Note that the dividend yield follows from output remaining after internal investment, and the capital gains rate follows from \( \frac{dk(t)}{dt}/q_t \), where \( q_t \) follows from (25).
optimal consumption rates are simply equal to the discount rates times agents’ wealth levels (Merton, 1992, ch.4).

Intermediaries choose portfolio weights \( x_t \) and \( 1 - x_t \) on capital and money, respectively, to solve

\[
\max_{x_t} x_t r_t^K + (1 - x_t) r_t^M + \lambda \log \left( x_t (1 - \phi) \frac{\tilde{q}_t}{q_t} + (1 - x_t) \frac{\tilde{p}_t}{p_t} \right),
\]

which yields the following first-order condition:

\[
r_t^K - r_t^M + \lambda \frac{(1 - \phi) \frac{\tilde{q}_t}{q_t} - \frac{\tilde{p}_t}{p_t}}{x_t (1 - \phi) \frac{\tilde{q}_t}{q_t} + (1 - x_t) \frac{\tilde{p}_t}{p_t}} = 0.
\] (29)

Similarly, households choose portfolio weights \((x_t, 1 - x_t)\), \( x_t \in [0, 1] \) to solve

\[
\max_{x_t} x_t r_t^K + (1 - x_t) r_t^M + \lambda (1 - \phi) \log \left( x_t \frac{\tilde{q}_t}{q_t} + (1 - x_t) \frac{\tilde{p}_t}{p_t} \right) + \lambda \phi \log (1 - x_t),
\]

which yields the first-order condition of households

\[
r_t^K - r_t^M + \lambda (1 - \phi) \frac{\tilde{q}_t}{q_t} - \lambda \phi \frac{1}{1 - x_t} = 0.
\] (30)

D. Market Clearing

Let \( N_t \) be the total net worth of intermediaries. Then the aggregate net worth of households is \((p_t + q_t)K_t - N_t\). As stated above, consumption rates are simply equal to discount rates times net worth and hence independent of investment opportunities. In other words, there is no precautionary saving. The market-clearing condition for output is therefore

\[
(a - \omega_t)K_t = \rho N_t + r ((p_t + q_t)K_t - N_t).
\]

The market-clearing condition for capital follows from equating the aggregate value of capital in the economy to the total value of capital held by intermediaries and households

\[
q_t K_t = x_t N_t + p_t ((p_t + q_t)K_t - N_t).
\]

Denote by

\[
\theta_t = q_t / (p_t + q_t)
\]
the fraction of the economy’s wealth that is invested in capital, and by
\[ \eta_t = \frac{N_t}{(p_t + q_t)K_t} \]
the fraction of total wealth owned by intermediaries. Then the market-clearing conditions can be rewritten as
\[ \frac{a - \iota_t}{p_t + q_t} = \rho \eta_t + r(1 - \eta_t) \quad \text{and} \quad x_t \eta_t + x_t (1 - \eta_t) = \theta_t, \tag{31} \]

In the absence of shocks, the law of motion of the net worth of intermediaries, given their optimal portfolio and consumption choices, is
\[ \dot{N}_t = (x_t r^K + (1 - x_t) r^M - \rho) N_t. \tag{32} \]

In the event of a shock, net worth of intermediaries changes to
\[ \tilde{N}_t = \left( x_t (1 - \phi) \frac{\dot{q}_t}{q_t} + (1 - x_t) \frac{\dot{p}_t}{p_t} \right) N_t. \tag{33} \]

Equations (32) and (33) also imply the evolution of net worth of households which is given by \((p_t + q_t)K_t - N_t\).

E. Equilibrium

BS identify two important benchmark cases. First, the pure money regime in which intermediaries have zero net worth and \(\lambda > 0\). In this scenario, saver households hold outside money only and all capital, so there is no inside money in this regime. Second, the frictionless regime in which agents can perfectly insure themselves against redistributional shocks, i.e. \(\lambda = 0\). In this scenario, the financial friction becomes obsolete as intermediaries do not need buffers to absorb any risk. This makes both capital and money risk-free and leads to unimpeded creation of inside money by intermediaries. Intermediaries are well capitalised and are able to lend freely.

The equilibrium path of the economy lies between these two regimes. In the absence of monetary policy, a negative macroeconomic shock decreases the net worth of intermediaries and lowers their capacity to mitigate financial frictions. The price of capital decreases, while the price of money increases. This leads to a lower money multiplier (total money to outside money), as the economy drifts towards the money benchmark. As the economy recovers, intermediaries are able to create more inside money, increasing the money multiplier and lowering the value of outside money. After the recovery, the economy moves towards the frictionless benchmark.

To characterise an equilibrium, one has to find prices and allocations for any history of previous shocks, such that all agents maximise utility and all markets clear. More formally, a definition of an equilibrium is the following.
Functions $\theta$ intermediaries drops by a factor in the absence of shocks. In the event of a shock, the wealth share of $\iota$ and $\eta$ time, to prices $p_t$ and $q_t$, allocations $(x_t, 1 - x_t)$, $(z_t, 1 - z_t)$ and $(\epsilon_t, \gamma_t)$, as well as wealth levels $(N_t, (p_t + q_t)K_t - N_t)$, such that

(i) all markets (for money, capital and consumption goods) clear,

(ii) all agents choose portfolio allocations and consumption rates to maximise utility,

(iii) and wealth levels of agents satisfy their budget constraints.

From the laws of motion of the aggregate wealth levels and the equilibrium conditions, one can derive the drift of $\eta_t$, which is

$$\mu^\eta_t = \frac{\dot{\eta}_t}{\eta_t} = x_t \frac{1 - \tau(a - \iota_t)}{q_t} + (1 - x_t) \left( \frac{\tau(a - \iota_t)}{p_t} + \iota_t + (x_t - \theta_t) (\mu^q_t - \mu^\iota_t) - \rho \right)$$

in the absence of shocks. In the event of a shock, the wealth share of intermediaries drops by a factor

$$\frac{\dot{\eta}_t}{\eta_t} = x_t (1 - \phi) \frac{\delta_t}{\theta_t} + (1 - x_t) \frac{1 - \theta_t}{1 - \theta_t}.$$  

Finally, it is useful to summarise all equilibrium conditions in the following proposition, including scaled variables only instead of aggregate variables.

**Proposition.** In a Markov equilibrium, the law of motion of the state variable $\eta_t$ is given by (34) and (35), where $q_t = \theta_t (p_t + q_t)$, $p_t = (1 - \theta_t)(p_t + q_t)$, and $\iota_t, p_t + q_t$ satisfy

$$\theta_t (p_t + q_t) \Phi'(\iota_t) = 1, \text{ and } (r + (\rho - r) \eta_t)(p_t + q_t) = a - \iota_t.$$  

Functions $\theta_t = \theta(\eta_t)$, $x_t = x(\eta_t)$ and $z_t = z(\eta_t)$ are jointly determined by

$$\frac{1 - \tau(a - \iota_t)}{q_t} - \frac{\tau(a - \iota_t)}{p_t} - \iota_t + \theta'(\eta) \mu^\eta_t \eta_t$$

$$+ \lambda \left( 1 - \phi \right) \frac{\delta_t}{\theta_t} - 1 - \frac{\theta_t}{1 - \theta_t} \right) \frac{\eta_t}{\eta_t} = 0, \text{ and }$$

$$\frac{1 - \tau(a - \iota_t)}{q_t} - \frac{\tau(a - \iota_t)}{p_t} - \iota_t + \theta'(\eta) \mu^\eta_t \eta_t$$

$$+ \lambda(1 - \phi) \frac{\delta_t}{x_t - \eta_t} + (1 - x_t) \frac{1 - \theta_t}{1 - \theta_t} - \lambda \frac{\dot{\phi}}{1 - x_t} = 0.$$
Proof. See BS.

The proposition from above implies a first-order ordinary differential equation for \( \theta(\eta_t) \), which can be solved numerically to find an equilibrium for given parameter values. The outline of the algorithm used for computations in the next section can be found in the appendix of BS.\(^28\) To facilitate computation, a degenerate investment function \( \Phi : \{0\} \to \mathbb{R} \) is assumed leading to some growth rate \( g \equiv \Phi(0) - \delta \) while output is produced at rate \( a \). This has some major computational advantages exploited in the MATLAB implementation of the algorithm used for computing solutions.\(^29\)

F. Simulation

Table 2 gives the parameter values used for computation. Recall that \( a \) is a productivity parameter, \( r \) is the discount rate of households, \( \rho \) the discount rate of intermediaries, and \( \lambda \) the intensity of the macroeconomic shock. The idiosyncratic probability of households losing their entire capital \( \phi \) can be interpreted as a measure for market liquidity, while the fraction of capital intermediaries lose in case of a shock, \( \phi \), quantifies exogenous risk faced by the intermediary sector. The parameters \( \tau \) and \( \xi \) are related to fiscal and monetary policy, respectively.\(^30\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.06</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>1</td>
</tr>
<tr>
<td>( a )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.001</td>
</tr>
<tr>
<td>( \phi )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \xi )</td>
<td>0.09</td>
</tr>
</tbody>
</table>

As a first example, consider an economy without monetary policy. Recall that the fraction of the net worth of intermediaries \( \eta_t \) is the state variable of the economy, which determines the value share of capital \( \theta(\eta) \) on an interval \([0, \eta^\ast] \), where \( \eta^\ast \) is the stochastic steady-state. Figure 7 gives the equilibrium on the whole interval \([0, \eta^\ast] \). Recall that \( \mu_\eta \), which can be seen as the solid line in the bottom left panel, is the drift of the net worth of intermediaries from equation (34). The drift is monotonically decreasing towards zero over

---

\(^28\)The idea behind the algorithm is as follows: first, an initial value \( \theta(0) = (1 - \tau)r/(r + \lambda\phi) \) (which corresponds to the pure money regime) is defined on a small interval. Then the derivative \( \theta'(\eta) \) can be computed on that interval. Next, the system has to be solved for \( \eta \) along the tangent line for a small step size. This is accomplished by iteratively executing the following steps: (i) compute prices (for a given \( \eta \) which is initially close to 0), (ii) search the segment where \( \eta \) jumps to after a shock (by computing price jumps and portfolio allocations, given an initial guess for \( \hat{\eta} \)), (iii) compute \( \eta \) and \( \theta \) after the jumps using linear interpolation, (iv) compute \( \theta' \), and the drift of \( \eta \) and (v) increase \( \theta \) accordingly.

\(^29\)I am very grateful to Markus Brunnermeier and Yuliy Sannikov for providing me with a preliminary version of their code.

\(^30\)Extensive discussion of different parameter constellations is omitted and can be found in BS.
the whole interval, indicating that the economy is moving towards the steady state and staying there as long as there are no shocks. The dotted line in the bottom left panel gives the jump in $\eta$ once a shock arrives. Note that $\eta$ jumps down as some of the capital financed by intermediaries is stolen by other households. Importantly, the jumps are smaller than the drift, implying that the system tends to return to $\eta^*$, whereas the margin between drift and drop narrows.

The top left panel shows that $\theta(\eta)$ increases as the net worth of intermediaries rises, suggesting that capital becomes more valuable. The top-right panel shows the equilibrium allocations of capital to intermediaries $\psi_t = \eta_t x_t / \theta_t$. For a sufficiently large net worth of intermediaries, all physical capital is financed by intermediaries. The middle panels show the endogenous evolution of the prices of physical capital $q$ and money $p$. In the event of a negative shock money appreciates while capital depreciates.

When financial intermediaries are hit by a shock, they are forced to shrink their balance sheets. A decrease on the asset side of intermediaries means that they issue less inside money. Borrowers are forced to reduce their investment which decreases the demand for capital and thereby depresses the price of capital. A decline in the price of capital means that the value of the asset side of intermediaries is declining further. This liquidity spiral increases the effect of the shock on the asset side of intermediaries. In addition, by creating less inside money, the overall money supply decreases, but the value of outside money increases, i.e. deflationary pressure arises. Deflation increases the real value of the liabilities of intermediaries and hence negatively affects their net worth, giving rise to a Fisherian deflationary spiral. This means a negative shock affects both sides of intermediaries balance sheets, thus amplifying the initial shock. The size of the jumps of the prices of money and capital depends on the location of the equilibrium (middle-left panel).

Note that the money multiplier slightly decreases near the steady state. This is due to the fact that at this point intermediaries are already financing all of the capital in the economy, i.e. $\psi_t = 1$. As their net worth continues to increase towards the steady state, intermediaries need relatively less funds from savers to provide all the necessary inside money.

Interestingly, even though the level of exogenous risk (i.e. the fraction of capital that intermediaries lose in the event of a shock) is very low, endogenous risk is very high in the region where intermediaries are forced to reduce their assets, i.e. $\psi_t < 1$ and savers are forced to hold capital. This is due to the fact that low exogenous risk induces intermediaries to build up high leverage which can be sustained near the steady state. If the economy is slightly further away from the steady state, a deep crisis with the aforementioned effects follows. This gives rise to a volatility paradox: the system appears stable near the steady state due to low exogenous risk, but as soon as it move slightly away, high levels of endogenous risk emerge (Adrian and
Market liquidity is directly related to the relative advantage of intermediaries in monitoring borrowers. If the friction that savers face is lower, meaning that the valuation of capital is closer to the first-best value achieved by intermediaries, then prices of both capital and money react less in the event of a shock. With endogenous risk on the asset side and a deflationary spiral on the liability side of intermediaries, market illiquidity strongly amplifies the inefficiencies created by financial frictions.

Fiscal backing of money dampens the effect of the deflationary spiral, as it ensures that the value of money is strictly positive in equilibrium. For \( \tau = 0 \) the steady-state value of money would be zero, thus making it a pure flight to safety asset in crisis times. This would lead to an excessively large money multiplier, as agents do not have an incentive to hold outside money.\(^{31}\) Since, in this case, the wealth of intermediaries would be very high, the deleveraging process after a shock would be followed by an extreme increase in the value of money. By setting a strictly positive \( \tau \), a much more

\(^{31}\)Brunnermeier and Sannikov (2012) also note that deviating from the assumption of log-utilities introduces a precautionary savings motive, leading agents to hold money for safety considerations at all times.
stable equilibrium can be ensured.

Introducing monetary policy in terms of an interest rate rule will affect the price of money. In the following, the central bank is assumed to set the interest rate according to \( i_t = \xi \eta_t \), where \( \xi > 0 \) serves as a weighting parameter. Monetary policy affects the nominal return on money and how much intermediaries pay to depositors. As the interest rate is higher, as the net worth of intermediaries increases, the value of money rises due to this policy. As a consequence, the earnings of intermediaries fall and their steady-state wealth share decreases. Figure 8 shows the effect of introducing this interest rate policy.

\[ \begin{align*}
\theta(\eta) & \quad \psi(\eta) \\
\eta & \quad \eta \\
\text{jumps in } q \text{ and } p \text{ after shock} & \quad \rho \text{ and } q \\
\text{drop and drift in } \eta & \quad \text{money multiplier}
\end{align*} \]

**Figure 8:** Equilibrium for baseline parameter values (with simple monetary policy).

The effect of the simple interest rate rule is very limited in this model. The inefficiencies created by the financial frictions cannot be sufficiently mitigated through the interest rate channel affecting expected future interest rates. While the wealth distribution between intermediaries and households is shifted in favour of the latter, endogenous risk persists at high levels. This is why BS introduce perpetual long-term bonds issued by the central bank. The monetary authority then controls the short-term interest rate and the amount of outstanding long-term bonds paying a fixed interest rate.

\[ ^{32}\text{Note that the net worth of intermediaries cannot be negative in this model, implying } i_t \geq 0.\]
in money. To maintain a constant ratio of outstanding bonds to money, the central bank buys bonds and sells money, or, in other words, performs open-market operations. As only intermediaries are assumed to hold long-term bonds, the redistributive character of monetary policy is even more pronounced.

When the short-term interest rate is lowered as a response to a fall in $\eta_t$, long-term bonds appreciate, recapitalising those intermediaries that hold them. This now compensates for some of the lost demand for money induced by the shock, leading to a lower appreciation of money in downturns. Intermediaries can use the bonds for (partially) hedging against the shock and creating more inside money by borrowing against them (which is equivalent to higher leverage). Despite lower intermediary net worth, the money multiplier is larger and all capital is still financed by intermediaries in steady state, as the risk associated with the deflationary spiral is now reduced. While the strain on the liability side of intermediaries (i.e. the endogenous risk of money) is now reduced, the asset side is still exposed to defaulting borrowers. However, those financial institutions insuring themselves with bonds benefit significantly in this scenario. Note that this policy redistributes towards more cautious rather than failing institutions.

Replicating the extended model with long-term bonds would go beyond the scope of this thesis. However, the main results in terms of system dynamics are captured in the above exposition.
VI DISCUSSION OF THE TWO APPROACHES

Both models presented above feature heterogeneous agents, with some of the agents taking out debt to finance their projects (i.e. consumption or investment). In the first model, households decrease their debt obligations in response to worsening aggregate lending conditions, thereby effectively destroying inside money. The effect of debt emerges through a credit demand channel as overindebted households cut down their consumption expenditure. The second model, on the other hand, operates through a credit supply channel, as financial institutions decrease the supply of inside money in response to macroeconomic shocks. Both models can therefore be linked to the process of money creation outlined in section II.

In what follows, the main insights are evaluated with respect to their policy implications and empirical relevance. Some references to relevant literature that has gained particular attention in light of the recent crisis are also enclosed.

A. Main Insights and Policy Implications

The main insight of the deleveraging model from section IV is that the ZLB poses a significant obstacle to optimal monetary policy. Dynamic deleveraging leads to an endogenous natural rate of interest that depends on monetary policy, thereby making crisis periods more persistent than in a scenario with a purely exogenous natural rate of interest. The monetary authority should be concerned with the redistributive character of the deleveraging process, regarding borrowers and savers. It could speed up deleveraging and shorten the time the economy spends at the ZLB.\textsuperscript{34}

As noted above, the deleveraging model presents a somewhat unconventional stance with regard to optimal monetary policy. As agents rationally anticipate the how long the economy will be at the ZLB, committing to lower interest for an extended period of time, followed by a sharp increase might actually lead to an output boom. This, however, will come at the cost of an inflation rate that exceeds the target.

The deleveraging model might also be used to argue in favour of fiscal policy. EK argue that since Ricardian equivalence does not hold, as some agents are debt-constrained, fiscal policy can have a large impact. Redistribution funds from savers to borrowers eases the strain on overindebted borrowers, thus slowing down the deleveraging process and leading to a

\textsuperscript{34}Hall (2011) also emphasises the role of of debt-overhang, high consumer commitment to service debt and increased financial frictions following the crisis. In an environment of low inflation, the real interest rate might easily be unable to adjust such that demand in other sectors is stimulated sufficiently to preserve full employment. Similarly, Guerrieri and Lorenzoni (2011) show how a credit crunch can lead to a recession through increases in precautionary savings and debt repayments.
faster recovery of the economy.\textsuperscript{35} Arguing in favour of fiscal policy when the economy is at the ZLB is supported by several other recent contributions, e.g. Christiano, Eichenbaum and Rebelo (2011), Eggertsson (2011) and Werning (2012). These these studies typically assume an exogenous shock to the natural rate of interest. In contrast to the model of section IV, the duration of the liquidity trap is therefore exogenous and there is no interaction between policy and the binding constraint.

However, the restriction of the model to a linear approximation of its non-linear counterpart implies only local properties in contrast to full system dynamics. Richter, Throckmorton and Walker (2014) and Gavin et al. (2014), for example, study global solutions of the New Keynesian model, however focussing on technology and preference shocks. They note that once capital is introduced the linear approximations differ significantly from the non-linear solutions. Fernández-Villaverde et al. (2012) also study non-linearities of the standard New Keynesian model around the ZLB. They argue that linear approximations might obscure non-linear interactions between the ZLB and the optimality conditions of agents, especially since the policy function of the interest rate is non-linear by definition. While finding support for a larger multiplier when the economy is at the ZLB, they conclude that the dynamics of the economy near the ZLB are only poorly approximated by conventional linearisation.

The model of money creation outlined in section V, on the other hand, captures full system dynamics. A crucial feature of this model is the volatility paradox, which states that low exogenous risk might lead to the build up of high levels of systemic risk, because low risk environments invite financial institutions to build up leverage.\textsuperscript{36} Strong non-linear amplification effects might emerge in response to a macroeconomic shock. Brunnermeier and Sannikov (2012) argue that the event triggering the crisis – a Minsky moment – does not necessarily have to be of major economic significance because there might be large spillover effects throughout the financial sector.

Liquidity and deflationary spirals amplify the effect of macroeconomic shocks through financial frictions. Endogenous risk emerges due to shock induced price movements, as financial institutions might drive down their asset positions and create less inside money. The impact of this deleveraging is stronger when most projects are financed via debt (i.e. the money multiplier is large). In this framework, monetary policy can mitigate the effect

\textsuperscript{35}Geanakoplos (2014) also emphasises that leverage is more important than the risk-less interest rate when it comes to macroeconomic stability. He argues that in case of a large decline in asset values, partial debt forgiveness might be better for everybody.

\textsuperscript{36}Fostel and Geanakoplos (2012b) also emphasise the pro-cyclicality of leverage. By providing a model with heterogeneous beliefs and endogenous leverage, they show that, in normal times, agents increasingly invest in technologies that become very volatile in bad times.
Discussion of the two Approaches

of the deflationary spiral by providing liquidity to the financial sector in the form of long-term nominal assets. By reducing the nominal interest rate in economic downturns long-term bonds appreciate, thereby benefiting institutions which hedged against the risk of borrowers.\footnote{Brunnermeier and Sannikov (2012) note that even though aggregate well-being might be improved, providing an additional source of insurance through perpetual bonds gives also rise to moral hazard opportunities. Excessive changes should therefore be addressed with macro-prudential regulatory measures.} Note that the model highlights how price stability can be increased by using a financial stability channel, as lower endogenous risk is associated with less deflationary pressure in the event of a shock.

B. Main Frictions

The deleveraging model is based on a New Keynesian framework that typically incorporates some form of wage or price stickiness to motivate an interest rate channel for monetary policy. In the deleveraging model, however, the crucial friction is now a risk-free level of debt, which serves as a financial friction and determines the total borrowing capacity in the economy. Variations in the given safe level of debt can either encourage borrowers to build up leverage, or trigger a deleveraging process in which borrowers find themselves in an overleveraged state. The induced deleveraging process drives down the natural rate of interest. If the economy is close to the ZLB, a liquidity trap might have a depressing effect on aggregate demand, even if monetary policy ensures full flexibility and output stabilisation.\footnote{However, the shock to the borrowing constraint is purely exogenous, and thus neglects the importance of credit as a facilitating role during the financial boom. Lax financing constraints might foster misallocation of resources, which ultimately leads to falling asset prices, expenditure cuts, and so on. Instead of arguing in general equilibrium terms, the economy might also be in an excess disequilibrium enabled by debt and capital stock overhangs (Borio, 2012).}

The main role of monetary policy is to overcome the price and the financial friction by influencing agents’ reactions to a shock. Importantly, the policy rule is fully known to agents who form consistent expectations and maximise their desired objectives. Furthermore, money serves mainly as a unit of account, which ensures that the interest rate rule is the leading policy instrument.\footnote{See King (2002) for an interesting treatment of money in conventional macroeconomic models.} This excludes money from serving other roles, such as protection against uncertainty. However, introducing the borrowing constraint leads to a substantial deviation from the complete markets benchmark. This is manifested in having a savings rate, a borrowing rate and a natural rate of interest all behaving quite differently in the case of a shock, and therefore preventing the economy from reaching full-employment immediately.

The model of money creation on the other hand, solely relies on financial frictions as its source of inefficiency, while money also serves as a store...
Again, financial frictions prevent agents from perfectly insuring themselves against macroeconomic shocks, which leads to shifts in the wealth distribution. As initial shocks are amplified through price movements, systemic risk arises endogenously. Since asset holdings are not symmetric across different groups of agents, monetary policy has asymmetric effects and hence a redistributive character. Monetary policy can mitigate endogenous risk and stabilise the economy by ensuring that funds are channelled to productive projects.

C. Link to Empirics

A growing strand of recent empirical contributions deals with leverage of both households and financial institutions. To underpin the relevance of the above models, it is useful to look over some of this literature.

Schularick and Taylor (2012) provide a long-run analysis of the dynamics of money, credit and output. They identify two financial eras, where in the pre-WWII era money and credit maintained a roughly stable relationship, while in the post-WWII era a divergence between money and credit supply emerged. They emphasise that the second era is associated with unprecedented financial risk and leverage. The authors also note that crises in the post-WWII era mostly avoided debt-deflation, since monetary policy seemed to have taken a more aggressive stance against deflation. Furthermore, they find support for a positive relationship between financial fragility and size of the financial sector, as well as that shocks in stock markets can be more problematic in economies with high degree of financialisation. Jordà, Schularick and Taylor (2013) complement this study by focusing on the link between private debt overhang and the business cycle. They find that countries with larger credit booms before the 2008 collapse are facing slower recovery than economies with smaller credit booms. Both studies emphasise that the data calls for the inclusion of financial factors in macroeconomic models.

Mian and Sufi (2011) focus on the effect of house prices on household debt. As the majority of US households already owned property before housing price boom, they benefited from the rising prices. The borrowing by these existing home-owners contributed significantly to the rise in US household leverage before the outbreak of the crisis. In addition, Mian and Sufi (2012) show that the drop in aggregate demand, following a shock to household balance sheets at the outbreak of the crisis, is associated with a sharp increase in unemployment in the first two post-crisis years. They also find that a drop in aggregate demand has been more pronounced in states with high household debt. Mian, Rao and Sufi (2013) then show that

\footnote{The role of money being more liquid than other assets is also explored in Kiyotaki and Moore (2012). In this framework, investment opportunities are randomly distributed and agents hold money in their portfolio because they expect to be liquidity constrained at some point in the future.}
households, following a shock to their net worths, significantly reduce their spending. Mian and Sufi (2014) further argue that the decline in housing net worth significantly contributed to the decline in employment in the US, especially in states with high household leverage and a strong tie to the non-tradable sector. Glick and Lansing (2010) further note that these patterns are not unique to the US, as industrial countries with the largest increases in household leverage also tended to experience accelerating housing prices, followed by significant drops in consumption once prices declined. The importance of household leverage and the aggregate demand channel in light of the recent crisis is also supported by Philippon and Midrigan (2011) for the US and Martin and Philippon (2014) for the Eurozone.41

Borio (2012) stresses the relevance of a financial cycle, which he defines as “[...] self-reinforcing interactions between perceptions of value and risk, attitudes towards risk and financing constraints [...].” Drehman, Borio and Tsatsaronis (2012) empirically identify the financial cycle as medium-run co-movement of property prices and credit. This type of cycle typically lasts for 16 years, so more than twice as long business cycles last. Peaks of the financial cycle are generally associated with financial crises. If contraction phases of the financial cycle coincide with downturns of the business cycle, then recessions are particularly severe.

Adrian and Shin (2010b) provide empirical evidence for the role of financial intermediaries’ balance sheets in determining aggregate liquidity. In times of rising asset prices, intermediaries find themselves in a situation with surplus capital in the sense that their leverage is too low. To exploit this excess capacity, intermediaries have to take on more short-term debt and look for potential borrowers, thereby expanding their balance sheets. They argue that continuing adjustments of strong balance sheets contributed substantially to the expansion of the sub-prime mortgage market.42

Summarising, there is substantial empirical evidence highlighting the impact of changes in debt levels on macroeconomic fluctuations, due to leverage decisions of both households and financial intermediaries. It has to be noted that the causality is not unambiguous. The deleveraging of the private sector might have triggered the distress of the financial sector, or vice versa. Once a financial crisis erupts, this reinforces the mechanisms illustrated in the above models.

41Similarly, Koo (2008) argues that Japan’s Great Recession can be attributed to balance sheet distress of firms that focus on paying back their debt rather than exploiting profitable investment opportunities.
42Adrian and Shin (2010a) also emphasise the importance of intermediaries’ balance sheet quantities with respect to monetary policy, as they might be indicative for the risk-taking behaviour of financial institutions.
D. Possible Shortcomings

As both models presented above are simplistic by nature, they lack several aspects that are related to the recent financial crisis. From an economic perspective, the ongoing discussion about the recent crisis is vast. From an open economy perspective, both approaches neglect the possibility of debt being denominated in foreign currency. Private-sector indebtedness in foreign currency makes the economy prone to a mechanism similar to Fisherian debt-deflation. As the domestic currency loses its value against the foreign currency this leads to an increase in debt obligations of firms and thus to a reduction in the borrowing capacity of firms. A subsequent fall in investment and output might lead to further depreciation and put further strain on firms indebted in foreign currency (Krugman, 1999; Aghion, Bacchetta and Banerjee, 2001).

As noted above, the deleveraging mechanism of section IV is very simple with a particular focus on the leverage decisions of households. Battacharya et al. (2011) put a greater emphasis on how learning and expectations about future investment profitability and growth influence borrowing decisions of financial institutions. Building on multi-period portfolio choice, they derive a framework with endogenous defaults and an endogenous borrowing rate that is determined by the expectations of the financial institutions. In their approach, over-optimistic expectations are a driving force behind overleveraging, i.e. their findings also provide support for Minsky’s hypothesis.

Geanakoplos (2010) also emphasises the role of leverage on the price of assets, but focuses on the ratio of collateral values to the down payment that must be made to buy assets. An agent applying for a loan has to negotiate the interest rate and how much he can borrow. The borrowing volume will be restricted by the lender to reduce the risk of default. In order to secure the repayment of a loan, lenders typically insist on collateral. How much collateral is required depends on how reliable the borrower seems and on the value of the collateral itself. During extended periods of low volatility, leverage might rise because lenders feel more secure. If some buyers manage to get more money by highly leveraged borrowing (i.e. getting a loan at a lower collateral rate), they will spend the loan on assets and drive their prices up. These highly leveraged borrowers are willing to spend more for the collateral since they can use it to borrow even more. So, borrowing rises as both the loan-to-value ratio and collateral prices increase, thus narrowing the margin of down-payment. When volatility or uncertainty increases, credit markets tighten and restrict the optimistic buyers from borrowing. As they now buy less assets, the assets fall into more pessimistic hands and hence decline in value. The decline in asset prices causes a drop in the wealth of the optimistic buyers who were highly leveraged and now have to sell assets to meet their margin. This leads to further declines in asset values and wealth levels and concludes in the same way that excessive deleveraging
Brunnermeier (2009) focuses on how losses in the mortgage market led to disruptions of the overall financial market. He identifies four core mechanisms that led from a mortgage crisis into an economy-wide financial crisis: first, the balance sheet effects of borrowers cause liquidity spirals. Second, the lending channel dries up as banks become increasingly concerned. Thirdly, bank runs erode capital. Finally, network effects emerge as financial institutions become increasingly interconnected. Risk diversification on credit networks becomes a source for financial instability, as emphasised by Battiston et al. (2012a,b), Acemoglu, Ozdaglar and Tahbaz-Salehi (2013) and Teteryatnikova (2014). This approach highlights that individual financial robustness is interdependent with the financial robustness of its counterparties, such that highly interconnected financial networks might be prone to high levels of systemic risk and default cascades.

The role of financial innovation in the build-up phase of the financial crisis is also frequently stressed, e.g. Gennaioli, Shleifer and Vishny (2012) and Fostel and Geanakoplos (2012a), as new financial instruments might have contributed significantly to the increase in asset prices and risk-taking. Brunnermeier and Sannikov (2012) further note that securitisation and other financial innovation can be beneficial in the sense that it helps investors to hedge idiosyncratic risk and reduce the cost of shocks. At the same time, however, it can amplify shocks throughout the whole economy via network effects. Pozsar et al. (2010) identify shadow banks as a crucial element in the context of interconnectedness of financial institutions and financial innovation. In a nutshell, shadow banks carry out traditional banking functions, but lack regulation and access to liquidity of conventional commercial banks, which means that a highly developed shadow banking sector might accelerate liquidity spirals. Conversely, shadow banking facilitates the expansion of liquidity provisions and rising asset prices, investment and output during boom times, but substantially decreases the resilience against macroeconomic shocks.43

The enumeration of shortcomings can be extended arbitrarily, e.g. by including developments in behavioural finance, moral hazard issues, principal agent problems and other rather microeconomic approaches dealing with financial crises. However, even though the focus on the roots and causes of the financial turmoil are diverse, all of them are in some way connected to debt.

43See, for example, Moreira and Savov (2014) and Meeks, Nelson and Alessandri (2014) for recent macroeconomic models incorporating shadow banking.
VII Conclusion

This thesis highlights two main channels through which debt might influence macroeconomic stability. The first channel is a demand channel that looks at the deleveraging behaviour of households. By adjusting its interest rate, the monetary authority aims to stimulate aggregate demand by influencing intertemporal consumption decisions of households. The ZLB on the nominal interest rate provides a substantial limit to the effectiveness of the monetary authority’s attempt as the fall in consumption of overleveraged households might be too large to be compensated for by other households at non-negative interest rates. The second channel looks at how the deleveraging process of financial institutions forces agents to channel funds away from their first-best use. This lowers asset prices, depresses investment and hence output. Conventional monetary policy cannot mitigate the endogenous risk generated in the economy, which arises due to the presence of financial frictions.

While the overleveraged state of households might be resolved by redistributive fiscal policy (e.g. taxing savers and transferring funds to borrowers) the effect of the deleveraging of financial intermediaries might be weakened by redistributive monetary policy (e.g. providing additional insurance opportunities for more cautious institutions in the form of long-term bonds). Both models show how the destruction of inside money might trigger economic dynamics that cannot be resolved by a central bank focusing on a simple interest rate policy. Moreover, while crisis periods in the deleveraging model emerge through the demand for credit, the model of money creation provides an explanation for instability through the supply of credit. Arguably, there might be a simultaneity problem when it comes to the recent recession, as the whole private sector deleveraged, thus reinforcing the mechanisms that lead to recessions.

The above approaches might serve as interesting baseline frameworks for addressing much richer questions with respect to the macroeconomics of debt. To give some examples, consider first the model of deleveraging: what happens if investment and capital are introduced, giving households an additional channel for consumption smoothing? What if agents are not able to rationally anticipate how long the ZLB is binding or how long monetary policy commits to a low interest rate? What if the model is solved for full system dynamics, i.e. taking non-linearities into account that might arise close to the ZLB? Concerning the model of money creation, there are additional open questions: what happens if the economy is at the ZLB or monetary policy follows some Taylor rule? What about relaxing the fiscal backing of money, i.e. providing another microfoundation of money? What if the model allows for a more active fiscal policy? At this point, it should be noted that both models presented above lack government debt as they
both assume government budgets to be balanced at all times. Soaring public
debt poses an additional problem when it comes to aggregate debt dynam-
ics, possibly leading to some interesting interactions with private debt, as
noted in the introduction.

The above exposition suggests that the role of debt as a driving force
in business cycle dynamics and as a potential cause for financial instability
should be addressed more thoroughly from a macroeconomic perspective.


References


Bundesbank. 2014. Geld und Geldpolitik. Schülerbuch für die Sekundarstufe II.


APPENDIX

A. Derivation of New Keynesian Aggregate Supply Equation

Using $y_t(j) = (p_t(j)/P_t)^{-\theta}Y_t$ and $y_t(j) = h_t(j)$, the profit maximisation problem of intermediate firms can be rewritten as

\[ E_t \left\{ \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T Y_T \left[ (1 + \tau) \Pi^{T-t} \left( \frac{p^*_t}{P_T} \right)^{1-\theta} - \frac{W_t}{P_T} \left( \frac{p^*_t}{P_T} \right)^{-\theta} \right] \right\} \]

\[ = \lambda_t Y_t \left[ (1 + \tau) \left( \frac{p^*_t}{P_t} \right)^{1-\theta} - \frac{W_t}{P_t} \left( \frac{p^*_t}{P_t} \right)^{-\theta} \right] \]

\[ + \mathbb{E}_t \alpha \beta \lambda_{t+1} Y_{t+1} \left[ (1 + \tau) \Pi \left( \frac{p^*_t}{P_{t+1}} \right)^{1-\theta} - \frac{W_{t+1}}{P_{t+1}} \left( \frac{p^*_t}{P_{t+1}} \right)^{-\theta} \right] \]

\[ + \mathbb{E}_t (\alpha \beta)^2 \lambda_{t+2} Y_{t+2} \left[ (1 + \tau) \Pi^2 \left( \frac{p^*_t}{P_{t+2}} \right)^{1-\theta} - \frac{W_{t+2}}{P_{t+2}} \left( \frac{p^*_t}{P_{t+2}} \right)^{-\theta} \right]. \]

The first-order condition with respect to $p^*_t$ can then be written as

\[ E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T Y_T (1 + \tau) \Pi^{T-t} \left( \frac{p^*_t}{P_T} \right)^{-\theta} \frac{1}{P_t} \]

\[ = \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T Y_T \frac{\theta}{\theta - 1} \frac{W_T}{P_T} \left( \frac{p^*_t}{P_T} \right)^{-\theta} \frac{1}{p^*_t}, \]

which yields

\[ \frac{p^*_t}{P_t} = \frac{\theta}{(\theta - 1)(1 + \tau) \mu} E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T Y_T \frac{W_T}{P_T} \left( \frac{P_T}{P^{T-t}} \right)^{\theta}. \]

Now let

\[ E_t = \mathbb{E}_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \lambda_T Y_T \frac{W_T}{P_T} \left( \frac{P_T}{P^{T-t}} \right)^{\theta} \]

\[ = \lambda_t Y_t \frac{W_t}{P_t} + \mathbb{E}_t \alpha \beta \lambda_{t+1} Y_{t+1} \frac{W_{t+1}}{P_{t+1}} \left( \frac{P_{t+1}}{P_t} \right) \]

\[ + \mathbb{E}_t (\alpha \beta)^2 \lambda_{t+2} Y_{t+2} \frac{W_{t+2}}{P_{t+2}} \left( \frac{P_{t+2}}{P_t} \right)^{\theta} \]

\[ + \ldots \]
\[= \lambda_t Y_t W_t + \alpha \beta \mathbb{E}_t \lambda_{t+1} Y_{t+1} \frac{W_{t+1}}{P_{t+1}} \left( \frac{\Pi_{t+1}}{\Pi} \right)^\theta \]
\[+ (\alpha \beta)^2 \mathbb{E}_t \lambda_{t+2} Y_{t+2} \frac{W_{t+2}}{P_{t+2}} \left( \frac{\Pi_{t+1}}{\Pi} \right)^\theta \left( \frac{\Pi_{t+2}}{\Pi} \right)^\theta + \ldots \]
\[= \lambda_t Y_t W_t + \alpha \beta \mathbb{E}_t \lambda_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} \right)^\theta, \]

and similarly
\[F_t = \lambda_t Y_t + \alpha \beta \mathbb{E}_t F_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} \right)^{\theta-1}.\]

Then the first-order condition from above can be rewritten as
\[\frac{p_t^*}{P_t} = \mu E_t.\]

From the definition of the aggregate price level
\[P_t = \left( \int_0^1 p_t(j)^{1-\theta} dj \right)^{1/\theta},\]
one can derive the law of motion of aggregate prices. Let \(S_t \subset [0,1]\) be the set of firms that do not revise their prices at time \(t\) while all other firms choose \(p_t^*\). Recall that firms index their prices with \(\Pi\). Then
\[P_t^{1-\theta} = \int_{S_t} (\Pi P_{t-1}(j))^{1-\theta} dj + (1-\alpha)p_t^{*1-\theta}\]
\[= \alpha (\Pi P_{t-1})^{1-\theta} + (1-\alpha)p_t^{*1-\theta},\]

which can be rewritten as
\[\frac{p_t^*}{P_t} = \left( 1 - \alpha \left( \frac{\Pi}{\Pi} \right)^{\theta-1} \right) \frac{1}{1-\alpha}.\]

The aggregate supply equation used in section IV immediately follows from the above results.

### B. Law of Motion of Price Dispersion

Note that the definition of price dispersion can be rewritten as
\[\Delta_t = \int_0^1 \left( \frac{p_t(j)}{P_t} \right)^{-\theta} dj \]
\[= (1-\alpha) \left( \frac{p_t^*}{P_t} \right)^{-\theta} + (1-\alpha)\alpha \left( \frac{p_t^{*1-\theta}}{\Pi P_t} \right)^{-\theta} + (1-\alpha)\alpha^2 \left( \frac{p_t^{*2-\theta}}{\Pi P_t} \right)^{-\theta} + \ldots\]
\[(1 - \alpha) \left( \frac{p_t^s}{P_t} \right)^{-\theta} + \alpha \left( \frac{\Pi_t}{\Pi} \right)^\theta \left[ (1 - \alpha) \left( \frac{p_{t-1}^s}{P_{t-1}} \right)^{-\theta} + (1 - \alpha)\alpha \left( \frac{p_{t-2}^s}{P_{t-2}} \right)^{-\theta} + \ldots \right] \Delta_{t-1} \left( \Pi_t \right)^\theta = (1 - \alpha) \left( \frac{p_t^s}{P_t} \right)^{-\theta} + \alpha \Delta_{t-1} \left( \frac{\Pi_t}{\Pi} \right)^\theta. \]

Combining this with the result for \( p_t^s/P_t \) from appendix A., this yields the recursive definition of the law of motion of price dispersion used in the main text.

C. Model Code for Linearised Deleveraging Model

```plaintext
% Linearised Deleveraging Model
% Christoph Scheuch, August 2014
% Define variables
var y cb cs hb hs pi i inot r b d w;
varexo ed;
parameters betas betab chi beta sigma varphi kappa target ...
i_st ib_st phi_pi phi_y bbar lambda phi sigma_d;
% Calibrate parameters
betas = 0.995;
betab = 0.99;
chi = 0.4;
beta = (1-chi)*betas+chi*betab;
sigma = 1;
varphi = 1;
kappa = 0.02;
target = 0.02/4;
i_st = (betas^(-1))*exp(target)-1;
ib_st = (betab^(-1))*exp(target)-1;
phi_pi = 9999999;
phi_y = 0;
bbar = 4/chi;
lambda = 0.14;
phi = 0.14;
sigma_d = 0.3;
% Specify model
model(linear);
y = chi*cb+(1-chi)*cs;
y = chi*hb+(1-chi)*hs;
cs(+1)-cs = sigma*(i-(pi(+1)-target));
cb(+1)-cb = sigma*(ib+lambda*(b-d)-(pi(+1)-target));
varphi*hb+(sigma^(-1))*cb = w;
varphi*hs+(sigma^(-1))*cs = w;
bbar/(1+ib_st)*(b-ib) = bbar/exp(target)*(b(-1)-(pi-target)) ...
+cb-y-hb+hs;
```
\begin{verbatim}
pi-target = kappa*y+beta*(pi(+1)-target);
ib = i+phi*(b-d);
d = d(-1)-ed;
inot = phi*pi*(pi-target)+phi*y*y;
i = inot;
r = -chi*(lambda+phi)*(b-d);
end;

%% Give initial values
initval;
y=0;
cb=0;
cs=0;
hb=0;
hs=0;
pi=target;
i=0;
ib=0;
b=0;
d=0;
r=0;
end;

%% Check
steady;

%% Shock and plot
shocks;
var ed = sigma_d^2;
end;

stoch_simul(order=1,nocorr,nomoments,irf=20);
\end{verbatim}

\section*{D. Code for Linearised Deleveraging Model at the ZLB}

\begin{verbatim}
% Deleveraging model at ZLB
% Based on Guerrieri and Iacoviell (2014)
% Christoph Scheuch, August 2014

clear
global M_ oo_
modnam = 'dd_model';
modnamstar = 'dd_model_ZLB';
% Define binding constraint
constraint = 'i<0';
constraint_relax = 'inot<0';
% Define shock and timing
irfshock = 'ed';
shockssequence = [0 0.3 ];
nperiods = size(shockssequence,1)+18;

%% Main Algorithm
[zdatalinear zdatapiecewise zdatass oobase Mbase_ ] = ... 
solve_one_constraine (modnam,modnamstar,...
constraint, constraint_relax,...
shockssequence,irfshock,nperiods);
\end{verbatim}
%% Label Results
for i=1:Mbase_endo_nbr
    eval([deblank(Mbase_endo_names(i,:)), ... 'noZLB=zdatalinear(:,i);']);
    eval([deblank(Mbase_endo_names(i,:)), ... 'wZLB=zdatapiecewise(:,i);']);
    eval([deblank(Mbase_endo_names(i,:)), ... 'ss=zdataass(i);']);
end
%% Post-processing of results
ilevel ss=ist;
ilevel wZLB=400*(i_wZLB+ilevel ss);
ilevel noZLB=400*(i_noZLB+ilevel ss);
iblevel ss=ib_st;
iblevel wZLB=400*(ib_wZLB+iblevel ss);
iblevel noZLB=400*(ib_noZLB+iblevel ss);
real level ss=ist;
real level wZLB=400*(r_wZLB+rlevel ss);
real level noZLB=400*(r_noZLB+rlevel ss);
p_i wZLB=pi_wZLB+target;
p_i noZLB=pi_noZLB+target;
b_wZLB=(1+b_wZLB)*100;
b_noZLB=(1+b_noZLB)*100;
Curriculum Vitae

Personal Data

Place & Date of Birth  Amstetten, December 6, 1989
Nationality  Austrian
Email  christoph.scheuch@gmail.com

Education

<table>
<thead>
<tr>
<th>Date</th>
<th>Degree</th>
<th>Institution</th>
<th>Course Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>09/2012</td>
<td>Mag.rer.soc.oec. (MSc equivalent)</td>
<td>Economics (research oriented)</td>
<td>University of Vienna.</td>
</tr>
<tr>
<td>10/2009</td>
<td>Bakk.rer.soc.oec. (BSc equivalent)</td>
<td>Economics</td>
<td>University of Vienna.</td>
</tr>
<tr>
<td>09/2012</td>
<td></td>
<td></td>
<td>Theses titles: <em>Inflexible Wages and Prices in Keynes’ and New Keynesian Theory</em> (supervisor: Karl Milford) and <em>Post Crisis - The Taxation of the Financial Sector</em> (supervisor: Margit Schratzenstaller-Altzinger).</td>
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Work Experience

<table>
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<tr>
<th>Date</th>
<th>Role</th>
<th>Institution</th>
<th>Responsibilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>03/2012</td>
<td>Teaching Assistant</td>
<td>Institute of Mathematical Methods in Economics, Vienna University of Technology</td>
<td>Teaching <em>Introductory Microeconomics</em> to undergraduate mathematical economics students.</td>
</tr>
<tr>
<td>06/2012, 03/2013</td>
<td></td>
<td></td>
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<tr>
<td>07/2014</td>
<td>Teaching Assistant</td>
<td>Department of Economics, University of Vienna</td>
<td>Preparing course material and problem sets for the undergraduate lecture <em>Macroeconomics and Inequality</em></td>
</tr>
<tr>
<td>10/2013</td>
<td>Teaching Assistant</td>
<td>Theresa Grafeneder-Weissteiner</td>
<td></td>
</tr>
<tr>
<td>01/2014</td>
<td></td>
<td>Department of Economics</td>
<td></td>
</tr>
<tr>
<td>05/2012</td>
<td>Research Assistant</td>
<td>Institute for Advanced Studies</td>
<td>Assisting in and conducting microeconometric research mainly dealing with topics concerning labour economics, tax-benefit systems and retirement schemes.</td>
</tr>
<tr>
<td>10/2013</td>
<td></td>
<td>Vienna</td>
<td></td>
</tr>
</tbody>
</table>
10/2012 | **Student Assistant** for Harald Fadinger at the Department of Economics, University of Vienna. Grading problem sets and homework of undergraduate economics and development economics students for the courses *International Macroeconomics* and *International Economics.*

10/2012 | **Teaching Assistant** at the Department of Economics, University of Vienna. Organising a self-organised lecture entitled *Growth, Distribution and Wage Policy* for graduate economics students by independently choosing topics and corresponding lecturers; grading problem sets and short essays.

03/2012 | **Teaching Assistant** for Karl Milford at the Department of Economics, University of Vienna. Preparing teaching material for the undergraduate lecture *Keynes for Beginners.*

03/2011 | **Teaching Assistant** for Immanuel Bomze at the Department of Statistics and Operations Research, University of Vienna. Teaching *Calculus* to undergraduate economics and statistics students.

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**Language Skills**

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<thead>
<tr>
<th>English</th>
<th>Fluent</th>
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<td>German</td>
<td>Mother tongue</td>
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<tr>
<td>French</td>
<td>Independent</td>
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**Computer Skills**

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<th>MS-Office</th>
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<tbody>
<tr>
<td>LaTeX</td>
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<td>Stata</td>
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<tr>
<td>EViews</td>
<td>Basic user</td>
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<tr>
<td>Matlab</td>
<td>Basic user</td>
</tr>
<tr>
<td>GNU/Linux</td>
<td>Basic user</td>
</tr>
</tbody>
</table>

**Additional Information**

**Scholarships**

*Performance Scholarships* for the academic years 2009/10, 2011/12 and 2012/2013, financed by Austrian Federal Ministry for Science and Research.

Member of the *Class of Excellence 2011/2012* at the Faculty of Business, Economics and Statistics.

*Windhag Scholarship* of Lower Austria for academic achievements in the academic year 2009/10.
Further Activities

Elected *Student Representative for Economics* at the Austrian National Union of Students of the University of Vienna from 2013 to 2015.

Editor, layouter and co-author of various issues of *Der Rote Börsenkrach*, a paper for economics students at the University of Vienna, from 2011 to 2013.

Elected representative of research assistants in the *Institute’s Conference of the Institute for Advanced Studies* in 2013.