Diplomarbeit

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Study of the Atmospheric Wake of Madeira Island

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List of Acronyms

**ARW** Advanced Research WRF

**a.s.l.** above sea level

**EUFAR** European Facility for Airborne Research

**ECMWF** European Centre for Medium-Range Weather Forecasts

**GPS** Global Positioning System

**INS** Inertial System

**MBL** Marine Boundary Layer

**MM5** Mesoscale Model 5

**MODIS** Moderate Resolution Imaging Spectroradiometer

**NCAR** National Center for Atmospheric Research

**NCEP** National Centers for Environmental Prediction

**NCL** NCAR Command Language

**NetCDF** Network Common Data Form

**NWP** Numerical Weather Prediction

**RMS** Root Mean Square Error

**SAFIRE** Service des Avions Français Instrumentés pour la Recherche en Environnement

**SST** sea surface temperature

**TSK** skin temperature

**UTC** Universal Time Coordonné

**WRF** Weather Research and Forecasting
Abstract

The 57 km long and 22 km wide NW-SE oriented island of Madeira lies in the eastern Atlantic, 950 km SW from the southern tip of Portugal. The island’s highest peak, Pico Ruivo, reaches 1862 m above sea level (a.s.l.). Due to its size, atmospheric stability structure and winds upstream, wake formation in the downstream region of this mountainous island is expected, in particular in summer when the most sustained winds below the trade wind inversion come from NE, the direction perpendicular to the island orientation.

In situ and remote sensing measurements were obtained during the i-WAKE airborne campaign (Aug - Sep 2010) in Madeira. The research aircraft ATR 42 of Service des Avions Français Instrumentés pour la Recherche en Environnement (SAFIRE) collected data both upstream and downstream of the island. In the event that was analyzed in this work, data along ten flight legs downstream was collected on September 2, 2010. Wake signals such as abrupt wind jumps at the flanks of Madeira and warm core eddies are evident in this dataset.

A series of high-resolution simulations using the Weather Research and Forecasting (WRF) model is carried out to investigate how well the observed wake can be replicated. At the beginning of the analyzed period (September 1-3, 2010), the atmosphere was continuously stratified changing into a shallow-water regime with a strong inversion below the mountain-top toward the end of this period. It turns out that discrepancies of the wind components and potential temperature along the observed and simulated flight legs are remarkably small, although the wake phenomenon is non-linear in nature.

To investigate the main source of vertical vorticity in the wake, a vertical vorticity budget analysis has been carried out on the simulation data. It appears that for both flow regimes surface friction, a true vorticity source term, is the main contributor to the vorticity tendency at the flanks of Madeira. The further evolution of the vortices, which detach from the island, is mainly influenced by the advection- and divergence terms. Additionally, a sensitivity test shows a strong dependence of the evolution of eddies on the sea surface temperature (SST) which is linked to the vertical stratification of the atmosphere. It turns out that the discrepancies between simulated and measured flight legs are smallest for a uniform increase of SST (from National Centers for Environmental Prediction (NCEP) analysis) by +1 K.
**Kurzfassung**

Die 57km lange und 22km breite NW-SE orientierte Insel Madeira liegt im östlichen Atlantik, 950km südwestlich von der Südspitze Portugals. Der höchste Punkt der Insel, der Gipfel des Pico Ruivo, liegt auf 1862m über dem Meeresspiegel. Durch seine Größe, die Struktur der atmosphärischen Stabilität und die Winde stromaufwärts ist die Ausbildung einer Wirbelschleppe oder Wirbelstraße im Lee der Insel häufig, vor allem im Sommer, wenn unter der Passatinversion die stärksten Winde aus NE wehen, der Richtung normal zur Ausrichtung der Insel.


Ein Sensitivitätstest zeigt eine starke Abhängigkeit der Entstehung der Wirbel von der SST (Meeresoberflächentemperatur) auf, welche mit der vertikalen Schichtung der Atmosphäre zusammenhängt. Es zeigt sich, dass die Übereinstimmung von Messung und Modellsimulation am besten ist, wenn die SST (von der NCEP Analyse) uniform um 1K erhöht wird.
Chapter 1

Introduction

Figure 1.1: Atmospheric vortex street downwind of Madeira on 21 Aug 2010

If one considers a viscous isotropic fluid in a tunnel with a certain velocity and an
obstacle that extends vertically throughout the fluid, the formation of a wake downstream
of the object can be observed. Wakes are zones of reduced momentum in a fluid directly
behind an obstacle and consist of sharp shear lines at the flanks downstream of the object. Depending on the Reynolds number, the wake can become unstable and break up into vortices to form a Van Kármán vortex street. In the atmosphere such phenomena can be observed on a much larger scale than in tunnel experiments appearing as striking strato
cumulus patterns like the one in Figure 1.1 downstream of Madeira.

First observations of island induced vortex streets were made in satellite images in the 1960’s downstream of Guadelupe (Bowley C. J. et al., 1962) and Madeira (Hubert and Krueger, 1962). Due to the scale of this phenomenon which is too small for the in situ weather observation grid and too big for an observer to be seen from the ground, vortex streets behind islands had not been discovered before the rise of the satellite technology. Since the second half of the 20th century the mechanisms of atmospheric wake formation have been extensively investigated. During the past 20 years also a few research campaigns were carried out to collect in situ and remote sensing data of island induced wakes (Hawaii’s wake (Smith and Grubišić, 1993), the wake of St. Vincent (Smith et al., 1997)). In situ data of island induced wakes are valuable because dynamical mechanisms giving rise to such phenomena can be studied from verified simulation data.

1.1 Mechanisms of Atmospheric Wake Formation

Since there are horizontal shear lines on the flanks of the zone of reduced momentum, vertical vorticity is present there. The aim of the theories presented in the next sections is to find the source of vorticity in the wake. Vorticity can only be generated by two terms in the vorticity equation (see e.g. Holton (2004)), the baroclinic term and the dissipation. All other terms are only redistributing the vorticity, yet they have influence on the overall evolution of the vortices.

\[
\frac{\partial \zeta}{\partial t} = \left( -u \frac{\partial \zeta}{\partial x} - v \frac{\partial (\zeta + f)}{\partial y} - w \frac{\partial \zeta}{\partial z} - (\zeta + f) \cdot \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right) \text{ Advection term} \\
- \left( \frac{\partial w}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial w}{\partial y} \frac{\partial u}{\partial z} \right) \text{ Divergence term} \\
+ \left( \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \text{ Friction term} \\
+ \frac{1}{\rho^2} \left( \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) \text{ Baroclinic term} \\
+ \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \text{ Tilting term} \tag{1.1}
\]

**Advection Term:** is responsible for the transport of pre existing vorticity with the wind. Pre existing vorticity is necessary for a contribution of this term.
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**Divergence Term:** is accountable for conservation of angular momentum. In case of a divergence of the wind field, vertical vortex columns get horizontally stretched and their rotation is reduced and vice versa. Vorticity has to exist.

**Tilting Term:** tilts horizontally oriented vortex tubes into the vertical or vice versa. This term is also a redistribution term.

**Baroclinic Term:** expresses changes of density along surfaces of constant pressure in the cartesian coordinate system. These density variations arise from horizontal temperature gradients and generate vertical vorticity. This is a true source term.

**Friction Term:** shows that the spatial variation of friction generates vorticity. Such gradients occur in regions of interaction of the atmosphere with an obstacle in form of surface friction. For the vertical component of the vorticity equation, surface friction at vertical surfaces is important. For Madeira’s case this would translate to higher friction at the steep coasts and less friction in the atmosphere at the same altitude at a certain distance to the obstacle. This is a true source term.

The description of the terms is valid for the vertical component of the vorticity equation.

### 1.1.1 Wake regimes

Strongly depending on the stratification and the flow speed as well as on the height and aspect ratio of the mountain, the airflow can attain different regimes. In Figure 1.2 these regimes are related to two parameters, the non dimensional mountain height $M$ which is a ratio between the height of the mountain $h_m$ and the fluid depth $H_\infty$ (height of boundary layer)

$$M = \frac{h_m}{H_\infty}$$

and the upstream Froude number $Fr_\infty$ which compares the speed of the fluid $U_\infty$ with the phase speed of shallow water waves with $g^* = g \frac{\Delta\Theta}{2}$ as reduced gravity.

$$Fr_\infty = \frac{U_\infty}{\sqrt{g^*H_\infty}}$$
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Figure 1.2: Regime diagram after Schär and Smith (1993a). Depending on the Froude number and the non-dimensional mountain height, 3 different wake regimes can be reached. From this diagram it follows that for a constant Froude number, a transition from a subcritical flow to an unstable wake with vortex shedding can occur only by decreasing the altitude of the inversion below the mountaintop.

For a continuously stratified atmosphere characterized by a constant Brunt-Väisälä frequency N and flow speed U and depending on the orientation and the horizontal aspect ratio of the obstacle, a transition from linear mountain waves even to vortex shedding can occur without a change of the atmospheric structure. See therefore Figure 1.3.

Figure 1.3: Regime diagram relating the non dimensional mountain height $\frac{Nh}{U}$, where $h$ is the maximum height of the obstacle with the horizontal aspect ratio $b/a$ of the mountain. A transition from linear mountain waves to vortex shedding can occur without a change of the atmospheric structure only by changing the aspect ratio of the obstacle. [From Lin (2007)]
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1.1.2 Baroclinically Generated Lee Vortices

This mechanism of vorticity production works in two steps, first the generation of horizontal baroclinic vorticity, and second the tilting of it into the vertical. Smolarkiewicz and Rotunno (1989) showed that the vorticity in a steady hydrostatic and inviscid system enters the linearized system at the second order of approximation. This theory will be presented in this section.

\[
\begin{align*}
  u_0 \frac{\partial u_1}{\partial x} &= -\frac{\partial \phi_1}{\partial x} \\
  u_0 \frac{\partial v_1}{\partial x} &= -\frac{\partial \phi_1}{\partial y} \\
  \frac{\partial \phi_1}{\partial z} &= b_1 \\
  u_0 \frac{\partial b_1}{\partial x} &= -N^2 w_1 \\
  \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} &= 0
\end{align*}
\]

Equations (1.2a) - (1.2e) are the first order approximated linearized equations which define a steady inviscid hydrostatic system with a constant wind in x direction. This closed system was used by Smolarkiewicz and Rotunno (1989). The indices in (1.2a) - (1.2e) stand for the order of approximation. To obtain the vertical vorticity equation (eqn. 1.3), the cross derivative of (1.2a) and (1.2b) has to be taken.

\[
\begin{align*}
u_0 \frac{\partial \zeta_1}{\partial x} &= 0
\end{align*}
\]

It is clear from eqn. (1.3), that for a first order approximation a generation of vertical vorticity is not possible. It must therefore enter in a higher order term. In the vorticity equation for a second order approximated system (eqn. 1.4), the tilting term appears on the right hand side and indicates the tilting of horizontally generated vorticity.

\[
\begin{align*}
u_0 \frac{\partial \zeta_2}{\partial x} &= \xi_1 \frac{\partial w_1}{\partial x} + \eta_1 \frac{\partial w_1}{\partial y}
\end{align*}
\]
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Since the system is inviscid, $\xi_1$ and $\eta_1$ have to be generated baroclinically, see the vorticity equations (1.5a) and (1.5b)

\[ \frac{u_0}{\partial x} \frac{\partial \xi_1}{\partial x} = - \frac{\partial b_1}{\partial y} \]  \hspace{1cm} (1.5a)  

\[ \frac{u_0}{\partial x} \frac{\partial \eta_1}{\partial y} = - \frac{\partial b_1}{\partial x} \]  \hspace{1cm} (1.5b)  

where $b_1 = -N^2 \delta_1$ is a buoyancy parameter with $\delta_1$ as vertical displacement of the perturbation. For an inviscid and adiabatic flow, the streamlines follow isentropic surfaces. Integrating (1.5a) and (1.5b) in x-direction (flow direction) and assuming that $\xi_1 = \eta_1 = 0$ at $x = \infty$ leads to

\[ \xi_1 = \frac{N^2}{u_0} \int_x^{\infty} \frac{\partial \delta_1}{\partial y} \, dx \]  \hspace{1cm} (1.6a)  

\[ \eta_1 = \frac{N^2}{u_0} \delta_1 \]  \hspace{1cm} (1.6b)  

The above equations describe how the tilting of isentropes along an obstacle gives rise to horizontal vorticity. This mechanism is illustrated in Fig. 1.4. For a flow from right to left directly above the surface, $\eta_1$ produced on the upslope cancels out when it gets advected over the obstacle because according to (1.6b) it depends directly on the displacement of the perturbation $\delta_1$ and declines to 0 downstream of the obstacle. $\xi_1$ on the other hand does not cancel out and gets advected downstream to form a vortex tube. The downward motion of the flow directly behind the obstacle tilts $\xi_1$ partially into the vertical. See therefore the schematic in Fig. 1.4. For a higher altitude, the perturbation $\delta_1$ looks more complex as can be seen in the 3d isentropic surface of an internal gravity wave in Figure 1.5.

To illustrate this more clearly, one can substitute $w_1$ in (1.4) with (1.2d) and use the relation $b_1 = -N^2 \delta_1$ to obtain an expression for vertical vorticity (1.7).

\[ \zeta_2 = \xi_1 \frac{\partial \delta_1}{\partial x} + \eta_1 \frac{\partial \delta_1}{\partial y} \]  \hspace{1cm} (1.7)  

With equation (1.7), it is straightforward to analyze the orientation of the vortices. The slope $\frac{\partial \delta_1}{\partial x}$ downstream of the mountaintop is positive. If $\xi_1 > 0$ then $\zeta_2 > 0$ and vice versa.
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Figure 1.4: Top view on an isolated mountain (gray shaded circle) with a flow from right to left (filled black arrow). Two thick lines across the mountain mark the locations of the corresponding vertical cross sections. Left cross section: The isentropic surface (dashed line) directly above the ground mirrors the topography. Air near the mountain surface is colder than the free air at the same height level. The cold air descends and vorticity is generated. Circular arrows indicate the horizontal vortices arising due to baroclinicity. The strongest baroclinicity is where the vertical cross section is placed. Top cross section: The orientation of vorticity due to upward and downward motion which is responsible for the tilting of the horizontal baroclinic vorticity into the vertical.

Furthermore Smolarkiewicz and Rotunno (1989) show that this theory is consistent with Ertels theorem that potential vorticity $PV = \vec{\omega} \cdot \nabla b = 0$ throughout the domain if no diabatic heating and dissipation is present, with $\vec{\omega} = (\xi, \eta, \zeta)$ as 3 dimensional vorticity vector and $b$ as buoyancy.
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Figure 1.5: Illustration of a 3D internal gravity wave behind an obstacle at the upstream height $Nz = \pi/2$. Upper part: Streamline in the center of the gravity wave as seen in the lower part. The circular arrows indicate the orientation of horizontal vorticity. Lower part: 3D isentropic surface in red above the topography in blue. Since Ertels theorem $PV = const$ in an inviscid and adiabatic flow is valid, the vortex lines (black solid arrow) must lie on an isentropic surface. The blue circular arrows indicate the orientation of vertical vorticity in that vortex line [From Epifanio (2003)]

1.1.3 Generation of Lee Vortices due to Energy Dissipation in the Flow

In contrast to the theory presented in the previous section, dissipation of energy is included now, so $PV = const$ is no longer valid. The theory to this mechanism of vorticity production was investigated by Schär and Smith (1993a) and will be presented within the next pages.

The basic idea is that due to energy dissipation in a hydraulic jump, a vorticity flux perpendicular to the flow gives rise to eddies. For a mathematical approach, the shallow water equations (1.8a/1.8b) for an initially irrotational and inviscid atmosphere would be the ones to use. Schär and Smith used a "pseudo inviscid system" (Schär and Smith, 1993a) where they allow energy dissipation in the region of hydraulic jumps although the
model is inviscid.

\[
\frac{Dv}{Dt} + g^* \nabla (h + H) = 0 \quad (1.8a)
\]

\[
\frac{\partial H}{\partial t} + \nabla \cdot (vH) = 0 \quad (1.8b)
\]

For our purpose, this system is converted into a dimensionless notation (see derivation in appendix A.1). In the next steps, a link between a vorticity flux and energy dissipation is derived.

\[
\frac{Dv}{Dt} + \nabla (h + H) = F \quad (1.9a)
\]

\[
\frac{\partial H}{\partial t} + \nabla \cdot (vH) = 0 \quad (1.9b)
\]

Equation (1.9a) is now the dimensionless momentum equation with an arbitrary force \( F = (F_x, F_y) \) added on the right side to represent dissipative processes, internal friction and viscous processes in the fluid. (1.9b) is the non-dimensional continuity equation. Applying \( k \cdot \nabla \times \) on (1.9a) and using the vector identity (A.11) leads to the vorticity equation

\[
\frac{\partial \zeta}{\partial t} + \nabla \cdot (v \zeta) = D_\zeta \quad (1.10)
\]

with \( D_\zeta = k \cdot (\nabla \times F) \) as dissipative term. With the vector identity (A.10) the dissipative term can be cast into divergence form

\[
D_\zeta = -\nabla \cdot J_N \quad \text{where} \quad J_N = k \times F \quad (1.11)
\]

Using the vector identity (A.11) on (1.9a) leads to

\[
\frac{\partial v}{\partial t} + \zeta k \times v + \nabla B = F \quad (1.12)
\]

where \( B = \frac{v \cdot v}{2} + H + h \) denotes the Bernulli function. Inserting (1.11) into (1.12) yields

\[
\frac{\partial v}{\partial t} + k \times (\zeta v + J_N) + \nabla B = 0 \quad (1.13)
\]
Taking now $k \times (1.13)$ leads finally to

$$\mathbf{v} \zeta + \mathbf{J}_N = k \times \left( \nabla B + \frac{\partial \mathbf{v}}{\partial t} \right)$$

(1.14)

For steady flow the equation above links now directly variations of the energy of the flow with a vorticity flux. This flux consists of two components, the advective part $\mathbf{v} \zeta$ and the dissipative part $\mathbf{J}_N = k \times \mathbf{F}$. The component of $\nabla B$ parallel to the flow must be linked with a purely dissipative vorticity flux because there is no velocity component perpendicular to the flow ($\mathbf{v} \zeta = 0$ across the streamlines). The component of $\nabla B$ perpendicular to the flow can instead consist of both components, advective part and dissipative part. See therefore Figure 1.6 where the orientation of the fluxes is indicated. If the flow is non rotating ($\zeta = 0$) in the upstream region then $\mathbf{v} \zeta = 0$ and will stay so until vorticity is generated. This states that vorticity has to be generated through dissipation along the hydraulic jump if $\mathbf{v} \zeta \neq 0$ in the downstream region. The theory presented above does not give information about how the vorticity is generated or in which term it enters the vorticity equation. Rotunno et al. (1999) showed that the tilting term of the vorticity equation can be linked with the potential vorticity flux. From that it is clear that the vorticity is generated by tilting of horizontal baroclinic vorticity into the vertical and enters the vorticity equation in the tilting term.

Figure 1.6: Schematic of vorticity fluxes in and downstream of a hydraulic jump. The thin solid arrows are streamlines in a steady non rotating flow of constant energy ($B = \text{const}$). The gray thick line indicates the region of a hydraulic jump which leads to a reduction of energy in the flow. The region of reduced $B$ is gray shaded. The resulting vorticity fluxes from eq. (1.14) are indicated as thick arrows (filled: advective flux, open: dissipative flux of $\zeta$). The sense of rotation of the resulting vortex columns is stated by the curled thin vectors. [From Epifanio (2003) after Schär and Smith (1993a)]
1.1.4 Generation of Vorticity by Surface Friction

Consider an isolated and steep obstacle with a rough surface and an inversion beneath the top of the obstacle separating a well mixed boundary layer underneath from a stratified layer above. If a mean flow is applied to this scenario, the layer below the inversion will be prohibited from going over the mountain and will be forced to go around, whereas the stratified layer above the inversion can go over the mountain and generate internal gravity waves. See a schematic in Figure 1.7a. Friction near the surface of the obstacle is higher than friction in the free atmosphere at a certain horizontal distance from the surface. From the friction term of the vertical vorticity equation (eq. 1.1) it is clear that a contribution of this term is directly linked to a spatial variation of the drag force. Figure 1.7b shows a schematic of such a situation. Wind speed decreases towards the surface, because the friction force there drags the flow. The generated vorticity gets instantaneously advected downstream from the flanks, resulting in two shear lines of counter oriented vorticity.

![Schematic](image)

(a) Schematic with an obstacle and an inversion which splits up the mean flow up into a 3D flow over the obstacle and a 2D flow which is forced to go around. [After Etling (1989)].

![Top view](image)

(b) Top view of the obstacle showing the horizontal wind profile of the zonal wind. Note that the velocity reduces to zero towards the surface of the obstacle. [After Hafner and Xie (2003)].

Figure 1.7: Two sketches illustrating the mechanism of frictional generation of vertical vorticity.

1.1.5 The Break Up of a Stable Wake into a Vortex Street

The concept of the transition from a stable wake to a vortex street presented in this section is based on the instability theory from Schär and Smith (1993b). For sufficiently
large momentum deficits the velocity profile in the wake will become unstable with respect to small perturbations that are advected from the upstream into the wake region. It is important to distinguish between two kinds of instabilities depending on the group velocity $c_g$.

1. **Convective Instability** For modes of this kind of instability $c_g \neq 0$ and the energy, which propagates with $c_g$, is advected out of the wake and hence cannot accumulate.

2. **Absolute Instability** Here $c_g = 0$ and the perturbation energy remains in the wake and can increase. Hence vortex shedding is expected.

The frequency of shedding can be calculated with the non dimensional *Strouhal Number*:

$$St = \frac{f_{shed} \cdot D}{U_\infty}, \quad (1.15)$$

where $f_{shed}$ is the shedding frequency, $D$ the characteristic length of the obstacle and $U_\infty$ the upstream velocity. The Strouhal Number is a function of the Reynolds number $Re$ and has values around 0.2 for typical atmospheric conditions. With known upstream windspeed and characteristic length of an obstacle, the frequency of shedding can be calculated.
1.2 The Island Madeira

1.2.1 Description of the island

The island Madeira, belonging to Portugal politically, is the biggest of three islands of an isolated archipelago, 950 km southwest from the southwest tip of Portugal in the Atlantic Ocean. The NW-SE oriented island is 57 km long and 22 km wide and has very steep and rough coasts of volcanic rocks and deep valleys cutting the high island across the mountain ridge. The highest peak of the mountainous island, Pico Ruivo, reaches 1862 m a.s.l. (see the topography below in Figure 1.8). Due to the location at 32.7° N, the Azores High has a strong influence on the local climate.

![Topography of Madeira](image)

Funchal is the capital of Madeira and is located at the south eastern bay of the island. This is the location from which the National Portuguese Weather Service launches radiosondes daily at 12:00 Universal Time Coordonné (UTC). Additionally, there is a grid of 14 weather stations around the island.

1.2.2 Climatology of Wake Events

Atmospheric wake formation leeward of Madeira is quite likely due to the
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(a) Wake directions.  
(b) Wake strength.

Figure 1.9: Histograms of wake direction and strength. Dataset from 2003 to 2010. [From Goger (2011)]

- isolated location of the archipelago
- oval shape of the island
- height of the mountain ridge
- strong and steady trade winds caused by the Azores High
- main wind direction from NE perpendicular to the orientation of the island
- strong trade wind inversions at low altitude caused by the Azores High

Wakes leeward of Madeira are documented in all main wind directions and in different strengths. Therefore Goger (2011) analyzed in her work satellite images of wake events occurring from 2003 to 2010 and documented the direction and strength. She found that on average 115 wake events per year are evident in stratocumulus cloud patterns. The number of events has a strong annual cycle with a maximum in the summer months from May to August. The reason for this distribution is suggested by the variability of the Azores High. In a climatological analysis of the Azores High Davis et al. (1996) show a strong seasonal dependence of the strength with a maximum from July to September and a center of the anticyclone 20° west of Madeira’s location. For that reason strong and steady winds from N and NE are expected during this period.

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The distribution of the number of wake events per month from 2003 to 2010 is asymmetric with an upward slope from February to August and a sharp drop from August to September, see Figure 1.9. SST could give answer to this interesting distribution feature because SST is lagging the temperature of land surfaces, a result of different heat capacities of land and ocean. In spring, when the strength of the Azores High increases, SST is cold compared to the atmosphere above and little convection is expected. Inversions are smoother because the large-scale descent has no counterpart (convection). In September, SST is at its maximum and increased convection in the marine boundary layer is expected since the sea surface is warmer than the atmosphere above. The increased buoyant activity below the inversion works against the anticyclonic descent and sharpens the temperature jump; it also increases the inversion height. This theory is suggested by the wake event count distribution in Figure 1.9 and is strongly supported by the time series of the inversion height in Figure 1.11, but still has to be further investigated.

The most likely wake direction is to the southwest of the island because of the steady trade winds due to the Azores High. The least frequent directions are to the north and east, which are related to rare pressure distributions and occur mostly in autumn and winter. Vortex shedding is also most likely in summer.

1.3 The i-Wake Campaign

1.3.1 Overview

To investigate physical mechanisms behind an atmospheric phenomenon by using model data, it has to be assured that the simulation is realistic. Hence high quality in situ measurements are extremely valuable for validating model output. To achieve this, an airborne measurement campaign was carried out to capture in situ data from a wake event in Madeira. The European Facility for Airborne Research (EUFAR) sponsored field campaign "i-Wake" consisted of two parts, (i) airborne in situ and remote sensing measurements of a variety of parameters, and (ii) a sailing ship measuring SST and vertical profiles of the ocean leeward of Madeira. The campaign took place from 25 Aug to 5 Sep 2010 in Madeira where four research flights were flown with different objectives listed in Table 1.1. The hosting airport of the ATR-42 research airplane was Porto Santo, the neighboring island of Madeira located 55 km NE of Madeira.
CHAPTER 1. INTRODUCTION

<table>
<thead>
<tr>
<th>Date</th>
<th>Nr. Flights</th>
<th>Objectives</th>
<th>Duration (hrs)</th>
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<td>4.3</td>
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<td>2010-08-30</td>
<td>1</td>
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<td>4.5</td>
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<td>2010-09-02</td>
<td>2</td>
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<td>2010-09-04</td>
<td>3</td>
<td>documenting the diurnal variation of the wake</td>
<td>3.1</td>
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Table 1.1: Research flight missions

The probability of wake formation in this period of time is quite likely as already discussed in the previous section (1.2.2). The wind histogram of the weather station Arriero in Figure 1.10 at 1590 m a.s.l. shows that the winds were rather stable from NE in the period from 1 Aug to 30 Sep 2010. Most of the time this station was above the trade wind inversion and indicates clearly that the direction of the trade winds is given by the position of the Azores High. In Figure 1.11 the seasonality of the strength of the Azores High is evident in the time series of the trade wind inversion height. Clearly, a tendency of the inversion height descending below the mountain ridge in the summer months is evident. Consistently to the wake climatology the trend from spring to summer is gentler than from summer to autumn. During the i-Wake period the inversion height was below the western plateau height most of the time. Due to this, flow splitting was present and the flow was forced around the obstacle. The wake event on 2 Sep 2010 was documented with two research flights and is analyzed in this work. The atmospheric conditions were favorable for wake formation on
1.3. THE I-WAKE CAMPAIGN

Figure 1.11: Time series of the inversion height in 2010. The dots indicate the inversion height, dashed lines the elevation of Pico Ruivo (1862m, red) and the western plateau (1590m, blue). This plot is based on sounding data. The altitude of the lowest jump of 6K in the spread was identified as the trade wind inversion height. [Sounding data source: http://weather.uwyo.edu/upperair/europe.html]

that day. Furthermore the data coverage in the wake region was best, yielding an excellent data set.

1.3.2 Wake Event on 2 Sep 2010

The synoptic conditions (Figure 1.12) were typical for the summer months with the high pressure belt around 30°N. The surface signal of the Azores High can be identified in Figure 1.12b NW of Madeira. The main wind direction from NE at the surface can be roughly estimated from the pressure contours. The vertical structure of the atmosphere changed from continuously stratified on 1 Sep 2010 at 12:00 UTC to a layered structure on 2 Sep 2010 at 12:00 UTC. From the operational sounding of Funchal (Figure 1.13), a sharp trade wind inversion at the height of 1000m a.s.l. can be identified. Towards 3 Sep 2010 this inversion sharpens even more. Conditions were therefore favorable for the formation of a strong wake. The air was almost saturated at the inversion level and suggests a possibility of a strato-cumulus layer in that region, which is evident in the satellite images (Figure 1.14a).
From the airplane, strong horizontal wind shear lines were easy to identify on the sea surface. Strong winds force the water waves at the ocean surface to break whereas in areas with weak winds the water surface is smooth. In Figure 1.14b a shear line at the western flank of the island is indicated by a red dashed line in the picture. One can see wave breaking of oceanic waves as white spots on the darker sea surface beyond the red
line where strong trade winds blew from NE and a smooth and brighter sea surface on the nearer side of the red line where no wave braking indicates an area of weak winds.

(a) Satellite image on 2 Sep 2010 at 12:00 UTC. [Source: www.sat24.com]

(b) Western shear line on 4 Sep 2010, view direction to N. Red dashed: indication of shear line [Source: Johannes Sachsperger]

Figure 1.14: Signals of a wake leeward of Madeira

The next chapters all refer to the wake event outlined in this section.
Chapter 2

Objectives

Numerical Weather Prediction (NWP) models improve continuously and the quality of forecasts rises. The horizontal resolution gets more and more reduced to resolve small scale features on the mesoscale. This can be sometimes misleading, because these features are very often not physical (of numerical nature) and wrongly interpreted. Since NWP models are very often integrated in forecasting systems of weather depending events like flood pre warning systems, systems to improve the cost efficiency of transports or event weather, models have to produce reliable forecasts. Not only for that reason it is important to know what the used model is capable of. In this work the mesoscale phenomenon of wake formation is simulated with the Weather and Research Forecasting (WRF) model and discussed to see how accurately WRF can simulate it.

Atmospheric wakes are nonlinear in nature and challenging for numerical models. The main aim of this work is to compare the results of WRF simulations of the atmospheric wake leeward of Madeira with airborne measurements of atmospheric parameters obtained during the i-Wake campaign. The measurements are compared to the model results along ten straight flight legs. From that one can see the performance of WRF in reproducing the wake of Madeira. Furthermore the mechanisms of the vorticity production of Madeira’s wake are of high interest in order to validate theories on wake formation. A vorticity budget analysis is done to see which terms of the vorticity equation (1.1) are contributing dominantly to the tendency of the vertical component of vorticity. Ongoing research on atmosphere - ocean coupling in wakes suggests an influence of atmospheric eddy formation on oceanic properties like SST and vice versa. The behavior of the vertical stratification of the atmosphere depending on SST is therefore also investigated.
with a sensitivity test.

Madeira’s atmospheric wake is quite prominent. Herein this work, the first analysis of it is presented.
Chapter 3

Methods and Data

High temporal resolution airborne in situ data and high resolution data from numerical simulations using WRF were used for the analysis of the atmospheric wake phenomenon in Madeira. In this section both data sources are explained in detail as well as tools that were used for data processing.

3.1 Observations

3.1.1 The Research Aircraft ATR-42 of SAFIRE

Figure 3.1: Picture of the research aircraft ATR-42 at the airport in Porto Santo. [Source: Johannes Sachsperger]
The research aircraft (see Figure 3.1) is operated by SAFIRE, a research flight operator for Météo France. It is a two propeller airplane scientifically equipped and modified for use in mid-troposphere studies. A variety of in situ and remote sensors allow a broad field of application. Inside the aircraft live monitors show the flight track with the current position and time series of the data.

<table>
<thead>
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<th>Sensor</th>
</tr>
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<tbody>
<tr>
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</tr>
<tr>
<td>Latitude</td>
<td>GPS</td>
</tr>
<tr>
<td>Longitude</td>
<td>GPS</td>
</tr>
<tr>
<td>Wind direction</td>
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<td></td>
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<td>- speed with respect to ground from GPS</td>
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<td>- air angles from 5 holes ATR radome</td>
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<td>- air speed</td>
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<td>Platinum resistance sensor</td>
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<td>- platinum resistance sensor</td>
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<tr>
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<td>Pyranometer</td>
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<tr>
<td>SST</td>
<td>3 down facing radiometers</td>
</tr>
<tr>
<td></td>
<td>- brightness temperature</td>
</tr>
</tbody>
</table>

Table 3.1: Aircraft sensors used for further analysis

Measurements with a sample rate of 1 Hz were done along a flight track which was defined after Mesoscale Model 5 (MM5) forecasts on the day before the flight mission. A list of the measured quantities available in the data files can be found in table 3.1. The high speed of the aircraft (≈ 100 m/s along flight legs) has an impact on the thermodynamic measurements. For that reason, the data had to undergo a quality control (done by SAFIRE) and a correction of the high speed influences. Wind data was obtained from
5 holes in the radome of the aircraft measuring the wind direction with respect to the aircraft. Wind speed and direction can be calculated from an inertial system which knows the attitude of the aircraft, the speed of the airplane with respect to ground (from GPS) and the air speed measured at the front of the aircraft. SST was obtained remotely with three down facing radiometers measuring the brightness temperature. For too high altitudes the SST signal at the radiometers was too weak. Hence the altitude was set to 100 meters a.s.l. (except for the soundings).

3.1.2 The Research Flights 44 and 45

For 2 Sep 2010 the MM5 model forecasts of a southwestern wake event were quite promising. Two research flights were planned and carried out. During the first one (number 44) in the morning, a spiral sounding at 33.7°N/−16.3°E and one flight leg upstream of Madeira were flown. The flight track of flight 44 is displayed in Figure 3.2a. The objective of this research flight was to check whether the forecasted conditions were real and wake formation was likely.

![Flight track 9:00 UTC to 10:00 UTC (Morning)](image1)

![Flight track 13:30 UTC to 18:00 UTC (Afternoon)](image2)

(a) Flight track Nr. 44  
(b) Flight track Nr. 45

Figure 3.2: Blue lines indicate the flight track, topography is gray shaded and red dots mark the location of spiral soundings.

[Topography data source: http://www.viewfinderpanoramas.org/dem3.html]
3.2 Numerical Simulations

3.2.1 Model Settings

To understand the mechanisms of vorticity production of Madeira’s wake, the mesoscale model WRF V. 3.3 from the National Center for Atmospheric Research (NCAR) using the Advanced Research WRF (ARW) core was used for experiments. WRF is a regional model and requires therefore parental data for initial conditions and boundary updates. Because of its high resolution, ECMWF operational analysis data with global 16 km horizontal grid spacing and 91 vertical levels was used for initial conditions and the boundary updates. ECMWF data already has a strong Marine Boundary Layer (MBL) inversion at 1000 m a.s.l. (at the same altitude as in the aircraft sounding) and due to this acts as a good data source for boundary updates. Surprisingly the initial time had an appreciable influence on the results. A sensitivity test on that showed that the initial time of 06:00 UTC on 1 Sep 2010 with 30 hours in advance to the time window of the research flight 45 leads to the most accurate results.

Since the physical processes which give rise to vertical vorticity in the wake are expected to be on the mesoscale, the initial state of ECMWF had to be downscaled by a mesoscale model. High resolution WRF simulations were set up to scale down from 16 to 1 km horizontal grid spacing. The downscaling was first tried with three two way nested domains (9 km, 3 km, 1 km). We found that instead one can directly scale down from 16 to 1 km with a single WRF domain with remarkable good results. The domain is shown in Figure 3.3a. To achieve feasible results, the boundary relaxation zone of the WRF domain had to be adequately large (32 grid points). Also the vertical resolution (58 levels with top at 50 hPa) was improved with additional model levels around the inversion height (Figure 3.3b) with the highest vertical resolution of 45 m. The high level density gives rise to the problem that the model gets more sensitive to numerical instabilities. Therefore the time
step had to be reduced to 5 seconds. The heights of the model levels are listed in appendix B.2. For detailed information about the model settings see appendix B.1.

(a) Horizontal dimensions are 500km x 500km. (b) Vertical level distribution over sea (without terrain). Note the higher level density around 1000m a.s.l.

Figure 3.3: WRF domain

3.3 Post-Processing Tools

Model output files are in the Network Common Data Form (NetCDF) format and have to be further manipulated. Model levels are terrain-following and were interpolated on constant z levels, since it is difficult to interpret atmospheric data on terrain following surfaces. Furthermore, interpolations along lines (flight legs) were necessary. Powerful post processing software is therefore needed. The most important ones are highlighted in this section.

3.3.1 NCAR Command Language

The NCAR Command Language (NCL) is developed by NCAR for data manipulation purpose. An interpreter runs NCL scripts and makes it easy to use it in shell scripts. It
comes with numerous predefined routines for data manipulation, interpolating and plotting. A number of map plotting routines can be directly used to plot model output. A huge number of options make NCL enormously flexible. Predefined WRF NCL routines are very user friendly.

### 3.3.2 Matlab

This huge mathematical tool was used for manipulation and plotting of observational data and maps. The Matlab script language is easy to handle and comes with powerful and very fast routines which allows an efficient manipulation of a huge amount of data.
Chapter 4

Results

4.1 Observed Wind Vectors

To show that a wake was present during the time frame which is analyzed in this section, the observed wind vectors of the aircraft are plotted in Figure 4.1. Clearly a wake region can be identified directly leeward of the Island where the vector direction is opposite to the mean flow. This states that the wake phenomenon is of a highly non linear nature, because the perturbations in the wind components are partially even larger than the mean flow from upstream. Note that this is not a temporal snapshot where positions of eddies can be identified but the whole obtained time series. Nevertheless the region of the wake is well defined.

Another feature which comes out more clearly later in the time series of the wind components in section 4.3.2 is the acceleration of the wind close to the flanks of Madeira. This is a result of mass conservation. The atmospheric conditions are pretty much similar to the sketch in Figure 1.7a where the upstream flow is divided into a 3D flow over the mountain and a 2D flow in the MBL which is forced around. The observed wind vectors of Figure 1.7a are the signal to the 2D flow. Since mass is conserved, wind speed has to increase at the flanks of the island to transport the additional air masses blocked by the mountain.
4.2 Sensitivity Test on SST

Initially this sensitivity test was motivated by discrepancies between simulated and measured potential temperature along the flight legs for a simulation with unchanged SST (0K forcing). The offset between the curves of 0.5 K (simulated theta along the legs was colder than the measurements) suggested an artificial warming of the MBL.

To investigate the sensitivity of the inversion to changing SST, four simulations with identical setup except for SST and skin temperature (TSK) were carried out. SST and TSK were changed uniformly over water surfaces (see the NCL script C.2 in appendix C). TSK is the radiation balance temperature at the surface and hence has to be changed too in order to get an impact of SST forcing on the potential temperature in the MBL. From the red curve in Figure 4.3 a height and strength dependence of the inversion on SST can be easily identified. The colder the SST, the lower and smoother is the trade wind inversion and vice versa. In addition to that the MBL gets warmer with increasing SST and TSK, see Figure 4.2. From these results, the best agreement between simulation and
measurements along the flight legs was found for the run with +1K SST and TSK forcing. An explanation of that behavior could be that the convection in the MBL, which works against the synoptic descending, is not so strong over colder sea surfaces and the inversion smoothes and lies at a slightly lower altitude. With increasing SST, the strength of convection rises too and mixes the MBL more strongly. This results in higher altitudes of the inversion and larger vertical gradients of potential temperature at the interface to the free atmosphere. Consequently stronger wakes are expected for higher SST and vice versa.

The fact that the simulation results are closer to the observations (see Figure 4.2) in the run with increased SST by +1K suggests that the SST analysis from the NCEP dataset which is used by ECMWF does not capture the mesoscale SST warming in the wake region which arises from low wind speeds and hence less downward mixing of surface water of the ocean.

These warm signals have an influence on the vertical stratification of the atmosphere and hence on the behavior of the wake. Studies of the Hawaiian wake (Hafner and Xie, 2003) or the Southern California Bight wake (Caldeira and P. Marchesiello, 2005) show that air sea interaction is one of the key factors for simulating wake dynamics properly. In the SST dataset of NCEP analysis no warm temperature anomaly is evident in the lee region of Madeira. In the high resolution (4km) Moderate Resolution Imaging Spectroradiometer (MODIS) aqua 8 day composite data set this warm region leeward is present although it is underestimated due to the smoothing of the 8 day composite. Using this data set as
Figure 4.3: Four vertical potential temperature profiles at the same upstream position (see Figure 3.2a for the location) on 2 Sep 2010 at 09:15 UTC where the red line shows simulation results, the blue line the measurements and the black line the vertical stratification of the parental ECMWF analysis data at 12:00 UTC on that day. Small circles indicate the height of model levels. The WRF setup is the same for all four simulations but SST and TSK are changed artificially by an uniform offset labeled on top of each figure.

SST source only slightly improves the RMS of the ECMWF nested WRF run. In Figure 4.4 a jump in measured SST by +2 K can be observed which is the double as high as the anomaly in the MODIS data set. From this figure it is also clear that the atmospheric wake and the ocean are influencing each other over the wind stress on the sea surface which is one of the main factors of air-sea interaction.
Figure 4.4: In regions of high wind speed (green) the SST signal (blue) declines and vice versa. The x axis shows the seconds since the start of the measurements along each flight leg. The correlation of wind speed and surface reaction in the surface water temperature is striking.

4.3 Comparison of Observations and Simulation

Many model simulations were carried out to achieve satisfying results. Additional model levels in the inversion helped to capture the strong MBL inversion. Furthermore the discrepancies between measured and simulated potential temperature along the flight legs suggested an artificial, uniform increase of 1K of the SST field. This increase had an effect on the evolution of the eddies and all comparisons showed a better reproduction of reality (see Figure 4.2). This section will therefore always refer to the run with increased SST of 1 K.

A problem one gets with the comparison between observational and simulation data is
that conditions change during measurements but from model data only a snapshot can be analyzed. The comparison along a flight leg is a good example for this issue. It took about 15 min to fly each of them. During this period of time the wake changes its shape. Model output on the other hand comes every 15 min as a snapshot of the atmosphere. As compromise, only one output file per leg is compared to the measured time series.

Before simulation data is used, it should be checked if the simulated features that appear are of a physical origin or numerical artifacts. Especially for a run where a downscaling from 16 km directly to 1 km is done, as it is the case, boundary artifacts are dangerous to be interpreted as real features. In the simulation that compares best with the measurements, MBL rolls were advected from the domain boundaries into the domain. To assure that these appearances are real, satellite images were analyzed and such MBL rolls were actually found in the area which the model domain covers. Furthermore two runs differing by 6 hours of initial time were performed. If these features were of a numerical nature, 6 hours of additional advection time should create huge differences between the two runs. The distribution of vertical velocity was almost identical, so these MBL rolls are appearing at a certain time and seem to be therefore of physical nature. Additionally, a ratio between buoyancy and wind shear approximated by \( \frac{(T_{\text{air}} - \text{SST})}{U_{\text{mean}}} \) was calculated (Todd D. Sikora, 2004) because at a certain threshold of this value (when convection gets too strong) a transition from boundary layer rolls to Bénard convection occurs. This was the case and again a physical nature is suggested.

These tests are evident enough to assume that these features are not a numerical artifact.

### 4.3.1 Vertical Stratification of the Atmosphere

Two spiral soundings were obtained on 2 Sep 2010. One upstream of Madeira in the morning and one downstream in the afternoon. Both soundings took about 15 min to be completed. In Figures 4.5 and 4.6 a comparison between measured and simulated sounding of WRF and ECMWF is presented. For both measured soundings (up- and downstream) the top is at 3200 m a.s.l.. Model data is extracted along a vertical line up to 3500 m a.s.l. at the closest grid point to the center point of the spiral sounding. ECMWF data is available every 6 hours and for comparison, the output at 12:00 UTC in between both sounding times is used. From that, one can see how well the atmosphere is represented in the parental data. The location of the upstream sounding is in the almost undisturbed atmosphere whereas the sounding downstream is in the disturbed wake region and due to
that more sensitive to changes in the model settings. ECMWF data does not represent the
topography of Madeira well. Discrepancies between WRF and ECMWF can therefore be
linked to mesoscale features in first order because wakes occur on the mesoscale and hence
require a high resolution to achieve accurate results.

**Upstream Sounding**

From the potential temperature profile of the upwind sounding in Figure 4.5, the MBL
inversion can easily be identified at an altitude of 1000 m a.s.l.. It is clearly evident that
WRF improved the shape of the ECMWF potential temperature profile. Nevertheless, the
inversion of the WRF simulation is not as sharp as in the measurements.

![Figure 4.5: Upwind sounding at 09:15 UTC at 33.65°N/16.3°W. See Figure 3.2a for the position. Blue: observations, red: WRF-ARW simulation at 09:15 UTC, black: ECMWF sounding at 12:00 UTC. Dots mark the altitudes of model levels.](image)

The wind profiles are strongly dependent on the position where they are taken from, even upstream where no orographic disturbances are advected. Stochastic convection is
present in the MBL and difficult to forecast. Although it is shallow convection, it has a
strong impact on the wind direction due to convergence and divergence at the surface.
WRF only slightly improves the wind profile of ECMWF below the inversion and mimes the parental data profile above it. Wind speed is overestimated by the models below 1000 m. The overestimation comes from the ECMWF boundary data which has an offset of about 3 m/s. Important is that the model does not reach another flow regime (diagram 1.2) than the measurements, to end up with a similar flow in the lee. In Table 4.1 below Froude number and nondimensional mountain height are calculated from the measurement data and the simulation data. Both reach the same flow regime, wake formation with vortices.

<table>
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<td>Wake formation with vortices</td>
</tr>
<tr>
<td>WRF simulation</td>
<td>0.40</td>
<td>1.6</td>
<td>Wake formation with vortices</td>
</tr>
</tbody>
</table>

Table 4.1: Link of measurements and simulation with flow regimes

**Downstream Sounding**

Figure 4.6: Downwind sounding at 15:45 UTC at 31.9°N/17.75°W. See Figure 3.2a for the position. Blue: observations, red: WRF-ARW simulation at 15:45 UTC, black: ECMWF sounding at 12:00 UTC. Dots mark the altitudes of model levels.
In contrast to the upstream sounding in the morning, the WRF solution underestimates the windspeed throughout the sounding downwind in the afternoon. The driving data exactly represents the wind speed at the surface and shows quite good results above the inversion but does not capture the minimum at 600 m a.s.l. of the measurements. The WRF simulation instead shows a decrease of wind speed with height in the MBL but with a minimum at 900 m. Anyway, the profile of the driving data gets improved. The fact that the simulation with the higher resolution captures the minimum suggests again that the coarser driving model does not resolve a mesoscale feature that is responsible for that signal in the sounding. Wind direction a parameter quite hard to predict since it is strongly depending on small scale features. It is suggested from the streamlines in Figure 4.7 that an eddy which passes the sounding location is responsible for the behavior of wind direction in the WRF profile. Already small differences in the location of the eddy in the real atmosphere and in the WRF atmosphere are enough to create huge differences in wind direction in the vertical profile. The two vertical profiles show that the WRF simulation captured the atmosphere quite well and it can be assumed that the simulated wake is reproduced closely to reality.

Figure 4.7: Streamlines on 2 Sep 2010 at 15:45 UTC at 600m a.s.l.. The red cross marks the location of the downstream sounding.
4.3.2 Comparison of Data along Flight Legs

Comparisons are done along ten straight flight legs, five of them downwind of Madeira and the other five upwind again towards the island. Model data was therefore vertically interpolated to the flight altitude at 100 m a.s.l. and interpolated along a straight line in the horizontal equidistant grid and transferred back to the latitude longitude grid which is not spatially equidistant. The numbering of the flight legs referring to the comparison plots and is shown in Figure 4.8.

**Potential Temperature**

Focusing on the potential temperature measurements in Figure 4.9, warm cores of eddies can be easily identified as warm peaks in the time series. The nearer the flight leg is to the Island (e.g. leg 1 and 10) the more intense these signals are because the generation of such vortices occurs at the steep coasts of the island. Warm cores of eddies can also be easily identified as red areas in the streamline plot with potential temperature in Figure 4.15a. The simulated flight legs mime the measurements not perfectly, but the shape of the observation curve is for the majority of the flight legs quite well reproduced. The nearer the leg is to the island, the better is the result. The simulated legs near the island tend to be slightly ($\approx 0.5K$) to cold and the warm spikes on the flanks are also not as strongly present as they are in the observations but nevertheless the results are satisfying since the
CHAPTER 4. RESULTS

Figure 4.9: Comparison of potential temperature (on vertical axis) along the ten flight legs. Blue: Aircraft observations. Red: WRF simulation output according to the time in the center of the flight leg.

shape of the time series is caught.

U Component

Although windspeed is overestimated by the model in the upwind sounding (Figure: 4.5) the discrepancies between observations and WRF simulation are remarkably small. Easily sudden wind jumps at the flanks of the wake can be identified in the island-near flight legs in Figure 4.10. Outside the wake region the zonal wind component is strongly negative and declines to zero in the wake. These wind jumps are responsible for the shear lines appearing on the sea surface like the one in Figure 1.14b where ocean wave braking occurs outside of the wake region and a almost flat sea surface is present inside. The gradients of windspeed at the edges of the wake region in flight leg ten are quite strong with \( \approx 5 \text{m s}^{-1} \text{km}^{-1} \). Along those sudden wind jumps, strong vorticity \( \mathcal{O}(10^{-3}) \) is present (see Figure 4.14). Small
4.3. COMPARISON OF OBSERVATIONS AND SIMULATION

Figure 4.10: Comparison of zonal windspeed U (on vertical axis) along the ten flight legs. Blue: Aircraft observations. Red: WRF simulation output according to the time in the center of the flight leg.

Disturbances advected from upstream can accumulate along these vorticity lines, the wake gets unstable and finally breaks up into a vortex street (Schär and Smith, 1993b). This is one explanation why the agreement of observations and simulation gets better towards Madeira. The nearer to the island the more stable is the wake.

V Component

Sudden wind jumps are also present in the meridional wind component dataset, see Figure 4.11. Again, the shape of the curve of measurements is well represented and for the majority of the simulated legs wind speed matches quite well the observations.
4.4 Vorticity Budget Analysis

To investigate the origin of wake vorticity a vorticity budget analysis has to be done. The terms in the equation for the vertical component of vorticity 1.1 are therefore discretized separately to see the amount of vorticity tendency produced by each of them. This helps to understand the mechanisms that give rise to lee vortices.

Since the results in the sections 4.3.1 and 4.3.2 suggest that the WRF simulation is very close to reality, conclusions about mechanisms may be drawn from simulation data.

4.4.1 Description of Calculation Algorithm

For discretization, centered differences in time and space are used for higher accuracy of the results.
1. From the simulation, fields (z, lat, lon, f, u, v, w, T, p) from three time-neighboring output files are loaded for the computation of tendencies.

2. Three interpolation height levels are specified for computation of vertical gradients.

3. u, v, w, T, p are interpolated from model levels to the specified z levels.

4. Wind components get spatially smoothed.

5. Relative vorticity is computed for two different times at the second height level to estimate the tendency for the time step in between.

6. The local vorticity tendency term is calculated.

7. The other terms except the friction term are discretized.

8. The friction term is computed as a residual of the other five terms in the vorticity equation 1.1.

An excerpt of the NCL script is presented in appendix C/C.1.

4.4.2 Results

The vorticity budget is calculated for a level in the middle of the MBL at 600 m to capture the processes at the half height of the vortices. The vertical distance between the three levels is 500m each. Because of the strong inversion 800 m below the mountaintop, the wake trapped in the boundary layer can be compared to a 2D wake where surface friction (strong gradients of friction near to the surface) at the flanks of the steep obstacle is the dominant source of vorticity (Ding and Street, 2003). The friction term in the vertical component of the vorticity equation is therefore expected not to be dominant compared to the other source term, the baroclinic term.
CHAPTER 4. RESULTS

Figure 4.12: Vorticity budget on 2 Sep 2010 at 15:30 UTC in 600m a.s.l.. Colored contours are the vorticity tendency $[s^{-1}/s]$ produced by each term. The black solid contours are coastlines. This budget analysis is purely based on simulation data and the used data fields are smoothed with a nine point averaging smoother applied 100 times.

Local Vorticity Tendency

From Figure 4.12 the estimated order of magnitude of the local tendency of vertical vorticity is $O(10^{-7})s^{-1}/s = O(10^{-4})s^{-1}/h$ what states that the strong lee eddies with $O(\zeta) = 10^{-4}s^{-1}$ which are present in the simulation (in a similarly smoothed wind field) can develop
within an hour. It should be noted that the tendencies at the steep flanks of Madeira are close to zero although strong relative vorticity is present there (see Figure 4.14).

**Friction Term**

Since the friction term is computed as a residual, arguments with it have to be done carefully as numerical errors can cause non-physical features. To prove that the term is physical, the causality of it has to be checked:

- dominant tendencies caused by friction gradients are at the steep coast of Madeira
- the signal at the western coast is stronger than at the eastern coast which is plausible because the western flank of the island is much steeper (higher horizontal gradients of friction component) than the eastern flank (can be seen from the topography in Figure 1.8)
- the sign of vorticity tendency is correct, positive vorticity tendency is expected at the western shore of Madeira and negative tendency at the eastern flank
- total vorticity tendency at both coasts is zero, which indicates that frictionally generated vorticity gets instantaneously advected away and cannot accumulate there
- the friction term is acting only at the interface of atmosphere and terrain

The dipole structure of the tendency of the friction term can be explained by looking at Figure 4.13. At the western flank a lee eddy causes upstream motion into the opposite direction to the mean flow (indicated by a blue circle) and hence results in giving rise to vertical vorticity of negative sign. A little to the NE, the opposite is the case where the mean flow hits the island and gives rise to vertical vorticity of positive sign due to surface friction at the steep coast. With these arguments, the plausibility of the friction term is given. Easily the dominance of the friction term in comparison to the second source term, the baroclinic term, can be seen in the vertical component of the vorticity budget in Figure 4.12.

From the un-smoothed relative vorticity distribution in Figure 4.14 two shear lines are evident as lines of vertical relative vorticity. Those are of opposite sign and originate from advected vorticity produced by surface friction at the steep coasts of Madeira. The shear lines are disturbed by perturbations advected from upstream. Depending on the group velocity of the perturbations (see section 1.1.5), some of them can stay in the shear line.
CHAPTER 4. RESULTS

Figure 4.13: Streamlines based on smoothed simulation data on 2 Sep 2010 at 15:30 UTC in 600 m a.s.l. The black solid contours are coastlines and the grey shaded area is the intersection of the plot surface with the terrain. The colored circles indicate areas where vorticity is generated by surface friction (blue: negative, red: positive).

and accumulate (absolute instability) to lead to a break up into a vortex street finally. The streamlines show a steady flow from NE disturbed by the island and as a consequence of it vortices in the lee of Madeira.

The concept of surface friction induced vertical vorticity tendency (see section 1.1.4) fits quite well to the simulation results and the observations.

Figure 4.14: Vertical component of relative vorticity in colors and streamlines at 600 m a.s.l. on 2 Sep 2010 at 15:30 UTC. The dotted area is the intersection of the plot surface with the terrain.
4.4. VORTICITY BUDGET ANALYSIS

Baroclinic Term

The scale of the wake phenomenon in Madeira is too small (meso scale) to expect a relevant contribution of vertically oriented baroclinically generated vorticity. The highest values of vertical baroclinic vorticity in Figure 4.12 are of an order of $O(10^{-8}) s^{-1}$/s and hence not relevant since the order of the highest values of the friction term is $O(10^{-6}) s^{-1}$/s. But as already discussed in section 1.1, horizontally oriented baroclinic vorticity can enter the vertical vorticity equation in the tilting term.

Tilting Term

According to the theories of Schär and Smith (1993a) and Smolarkiewicz and Rotunno (1989), which are presented in section 1.1, a signal in the tilting term can be expected to contribute significantly to wake formation. Such a signal can be found in the budget in Figure 4.12 but it has to be questioned if it originates entirely from horizontal baroclinic vorticity. In the comparison of the zonal and meridional components of the baroclinic term with the two parts of the tilting term which tilt those components into the vertical, a spatial matching of patterns can be found only for the red area at the western coast. For that reason it is assumed that the negative tendencies to the south of Madeira are not of a baroclinic origin. However the friction term is still stronger which supports the statement of Ding and Street (2003) that in cases of strong inversions below the mountain top, the wake is comparable to a 2D wake trapped underneath with friction as main vorticity source. The detaching of the MBL from the layer above is also evident in the vertical cross section in Figure 4.15 where a strong inversion is present in the lee- as well as in the lee region of the mountain ridge.

Advection Term

Advection plays a very important role in the budget. It mirrors the high tendencies of the friction term with an opposite sign and transports instantaneously the frictionally generated vorticity downstream. It is also important to note that the advection term has a non negligible effect in the free atmosphere detached from the island where it transports eddies further downwind. Furthermore small disturbances, which are easily to see in the shear lines in the un-smoothed relative vorticity distribution in Figure 4.14, are advected from upwind which can lead to a breakup into a vortex street as it is the case in the simulation (such disturbances are smoothed out before calculating the budget to remove
(a) Location and orientation of the cross section to the right is indicated by a thick green line. Streamlines highlight the direction of the flow and colors show the distribution of potential temperature at 600 m a.s.l.

(b) Vertical cross section across Madeira aligned parallel to the flow, wind comes from left to right. Colors are potential temperature and the thick solid line shows the height of the terrain surface.

Figure 4.15: The detaching of the MBL is evident in the vertical stratification with a neutrally stratified layer beneath the strong inversion and a stratified layer above.

Divergence Term

This term is responsible for the conservation of angular momentum or potential vorticity and is therefore dependent on already existing vorticity. The tendencies are more clear if we look at the expression of the term in the vorticity equation in (4.1).

\[
\text{DIV} = - (\zeta + f) \cdot \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
\]  

(4.1)

The order of magnitude of relative vorticity can be estimated from Figure 4.14 with \(\mathcal{O}(10^{-3})\) s\(^{-1}\) and is 2 orders higher than the coriolis parameter at 32°N where \(f = 7.7 \times 10^{-5}\) s\(^{-1}\). Hence the planetary vorticity can be neglected. In Figure 4.16 the divergence of the wind field is shown together with streamlines. Inserting this into equation (4.1) leads to an intensification of vorticity in convergent regions and vice versa in divergent regions which is consistent with the concept of conservation of angular momentum. Again, lee eddies are responsible for the dipolar shape.
Figure 4.16: Horizontal wind divergence and streamlines at 600m a.s.l. on 2 Sep 2010 at 15:30 UTC. The dotted area is the intersection of the plot surface with the terrain. This plot is based on smoothed wind data.

### 4.4.3 Flow Regime above the Inversion

Above the inversion the atmosphere is continuously stratified (see the cross section in Figure 4.15) and the flow regime for the upper layer can be estimated from the regime diagram in Figure 1.3. Assuming that the mountain ridge still towers 500m above the inversion, which is the case in the simulation, an inverse Froude number of \( Nh/U = 0.6 \) can be estimated from the vertical cross section in 4.15. Assuming that the aspect ratio of the mountain peaks is \( b/a = 1 \), the flow regime in the upper layer clearly suggests linear mountain waves and no wake. From the streamlines in Figure 4.17 it is clear that the wake is trapped underneath the MBL inversion since the vortex street signal in the streamlines has completely vanished in 1100m a.s.l.

Figure 4.17: Comparison of the motion of the flow in two different heights with streamlines on 2nd Sep 2010 at 15:30 UTC. Dotted areas are the intersection with the terrain.
4.5 Energy Budget at the Surface

To investigate the main impact of thermal surface forcing, a comparison of turbulent fluxes, (i) upward surface sensible heat flux and (ii) upward latent heat flux, is done in Figure 4.18. In regions of strong sensible heat flux the atmosphere is heated by the ocean and in regions of high latent heat flux the atmosphere becomes more humid due to evaporation of water from the ocean. Please note that the sensible heat flux is of a factor 10 lower than the latent heat flux. From that it is clear that a warmer ocean results mainly in evaporation of surface water into the atmosphere than in warming the boundary layer above. Interestingly, the SST forcing only has a far wake effect, whereas the near wake region is weakly affected. This issue has to be further investigated.

Figure 4.18: Comparison of the energy fluxes of two model runs of different forcing on 2 Sep 2010 at 15:45 UTC. The data from the plots of the left column is from the WRF run without SST forcing (second left sounding in Figure 4.3) and the plots of the right column correspond to the simulation with a forcing of +1 K (second right sounding in Figure 4.3). The upper row of plots shows the upward sensible heat flux and the lower plots the upward latent heat flux.
Chapter 5

Discussion

To extract useful information about mechanisms of wake formation from the model data, the fields have to be heavily smoothed since small numerical errors generate noise effects after an integration time of 30 hours. After the smoothing, plausible results appear. Nevertheless, two important open questions about the representativeness have to be answered. Firstly, why are the results plausible although the terrain is not well represented and secondly, why a downscaling of a factor of 16 from the driving data is done although it is not recommended for many applications.

5.1 Resolution Discrepancies of the Terrain

The data extracted from the simulation compares strikingly well to the measured in situ data along the flight legs, although the topography is not well represented. The highest peak of Madeira, Pico Ruivo, is at 1862m a.s.l. and located at 32.759°N/16.942°W whereas the highest peak in the model topography is at 1562m a.s.l. and located at 32.766°N/17.079°W which is 12 km to the West for a topography resolution of 30". Pico Ruivo falls through the 1 km grid spacing of the model topography. The reason why the results of the comparison between model and measurements are still remarkably good is that the inversion traps the wake entirely under 1000 m a.s.l. and the mechanisms giving rise to wake formation are not dependent on the topography above the MBL. Changes in the resolution of the model orography have an impact primarily on the height of the terrain, and not so much on the horizontal dimensions. Hence, the shape of Madeira underneath the stable inversion is sufficiently captured although the topography is not well resolved (300 m lower than in reality).
5.2 Downscaling Issues

In literature, mesoscale model simulations with a horizontal downscaling factor higher than 4 from the parental data are rare and for that reason the downscaling by a factor of 16 which is the case in the WRF simulation of Madeira’s wake herein this work has to be argued:

The first simulations showed that there is no gain of accuracy for simulations with nested domains. Model runs with 3 domains were carried out but boundary effects, resulting from the 2 way nested domains, effected the inner ones. Also a one way nesting was not beneficial. To save computational time, the outer domains were dropped and a single domain run set up. To achieve satisfying results, high resolution is necessary and first runs with a direct downscaling from ECMWF operational analysis data were successful. To tackle numerical artifacts resulting from the lateral boundaries of the domain, a large relaxation zone of 32 grid points was established to smooth such disturbances out. To assure that boundary artifacts arising from the spin up of the model are advected out of the domain before the time span of the measurement campaign is reached, the integration start was set to 30 hours in advance.

A reason why there are very few cases of model runs with a downscaling factor higher than 4 is that mesoscale models are very often used in regions of complex terrain throughout the whole domain to study orographical effects. In the case analyzed herein, the terrain is relatively simple. There are 900 km of undisturbed upstream flow over a flat surface before the island is reached. It makes not much of a difference if one grid cell over a flat surface is split into 16 with almost the same values. This makes a huge difference for flows over mountains which are not well represented in the coarser driving model but much better resolved in the nested mesoscale model. The integration can become unstable. In the simulation of Madeira’s wake, a very small time step of 5 seconds helps to overcome the sensitive model spin-up time without numerical instabilities in the region of the island and secondly leads to more accurate results.
Chapter 6

Conclusion

In this work, a case of an atmospheric wake event leeward of Madeira is analyzed by combining high resolution airborne in situ data, obtained during the field campaign i-Wake, with high resolution (1 km grid spacing) numerical real case WRF simulations. The remarkably well reproduced time series of measured parameters by the model (see section 4.3.2) allow further investigations of the driving mechanisms behind vorticity generation purely based on model data. For that purpose a vorticity budget is calculated where the vertical component of the full vorticity equation on a Cartesian grid is split into its terms and discretized to see the contributions to the tendency of the vertical component of vorticity. The budget clearly shows that the decline to zero of wind speed towards the steep coasts of Madeira is mainly responsible for generation of vorticity (see section 4.4.2) and caused by surface friction at the vertical coast surface. Baroclinic effects only play a minor role for this case. A climatology of the inversion height (see Figure 1.11) shows that, particularly in the summer months when the Azores High is strongest, inversions lower than Madeira’s mountain tops can be expected and hence similar dynamics can be made responsible for wake formation as they appear in the vorticity budget analysis in this work.

Furthermore, the response of the inversion to a change of SST was investigated with a sensitivity test with four simulations with identical setup except for SST and TSK. It appears that an increase of SST leads to a strengthening of the vertical potential temperature gradient and simultaneously to a lifting of the inversion height and vice versa for a decrease of SST. The best agreement of measured and simulated time series along the ten flight legs results for a simulation with a uniform change of SST by $+1K$. 
Appendix A

Derivations

A.1 Nondimensionalization of shallow water system

The inviscid non rotating shallow water equations have the form

\[
\frac{D\mathbf{v}}{Dt} + g^* \nabla (h + H) = 0 \tag{A.1a}
\]

\[
\frac{\partial H}{\partial t} + \nabla \cdot (\mathbf{v}H) = 0 \tag{A.1b}
\]

with \( \mathbf{v} = (u, v) \), \( h \) as topography height, \( H \) as fluid depth and \( g^* \) as reduced gravity. Starting with (A.1a), the dimension is \([m/s^2]\) and has to be scaled by a characteristic acceleration to achieve nondimensionality. From (Schär and Smith, 1993a) the definitions of characteristic scales are for the horizontal length \( a \), for the vertical length \( H_\infty \), for velocity \((g^* H_\infty)^{\frac{1}{2}}\) and for time therefore \( a \ (g^* H_\infty)^{-\frac{1}{2}}\). With these scales, non-dimensional
quantities and operators can be introduced

\[ \tilde{v} = \frac{v}{(g^*H_\infty)^{\frac{1}{2}}} \]  \hspace{1cm} (A.2a)
\[ \tilde{t} = \frac{t}{a (g^*H_\infty)^{-\frac{1}{2}}} \]  \hspace{1cm} (A.2b)
\[ \tilde{H} = \frac{H}{H_\infty} \]  \hspace{1cm} (A.2c)
\[ \tilde{h} = \frac{h}{H_\infty} \]  \hspace{1cm} (A.2d)
\[ \tilde{\nabla} = a \cdot \nabla \]  \hspace{1cm} (A.2e)

and a characteristic acceleration can be defined as

\[ acc_{\text{char}} = \frac{v_{\text{char}}}{t_{\text{char}}} = \frac{(g^*H_\infty)^{\frac{3}{2}}}{a \cdot (g^*H_\infty)^{-\frac{1}{2}}} = \frac{1}{a \cdot g^* \cdot H_\infty} \]  \hspace{1cm} (A.3)

By dividing (A.1a) by (A.3) one obtains the non-dimensional momentum equation of the system (A.4).

\[ \frac{Dv}{Dt} \cdot \left[ a \cdot \frac{g^*H_\infty}{(g^*H_\infty)^{-\frac{1}{2}}} \right] + \left[ a \cdot \frac{g^*H_\infty}{H_\infty} \right] \cdot g^* \nabla (h + H) = 0 \]  \hspace{1cm} (A.4)

Expanding the left term and canceling \( g^* \) in the right term yields to

\[ \frac{Dv}{Dt} \left[ a \cdot \left( g^*H_\infty \right)^{-\frac{1}{2}} \right] + \left[ a \cdot \frac{H_\infty}{H_\infty} \right] \cdot \nabla (h + H) = 0 \]  \hspace{1cm} (A.5)

Using the relations (A.2a)-(A.2e) in (A.5) leads to

\[ \frac{D\tilde{v}}{Dt} + \tilde{\nabla} (\tilde{h} + \tilde{H}) = 0 \]  \hspace{1cm} (A.6)

The dimension in the second equation in the system (A.9) is \([m/s]\) and has to be nondimensionalized with a characteristic vertical velocity (A.7).

\[ w_{\text{char}} = \frac{H_{\text{char}}}{t_{\text{char}}} = \frac{H_\infty}{a \cdot (g^*H_\infty)^{-\frac{1}{2}}} \]  \hspace{1cm} (A.7)
Dividing (A.9) by the characteristic velocity scale (A.7) yields to

\[
\frac{\partial H}{\partial t} \cdot \left[ \frac{a}{H_\infty \cdot (g^* H_\infty)^{\frac{1}{2}}} \right] + \left[ \frac{a}{H_\infty \cdot (g^* H_\infty)^{\frac{1}{2}}} \right] \cdot \nabla (vH) = 0 \tag{A.8}
\]

Using again the relations (A.2a)-(A.2e) gives the final form of the equation

\[
\frac{\partial \tilde{H}}{\partial \tilde{t}} + \nabla \cdot (\tilde{v} \tilde{H}) = 0 \tag{A.9}
\]

The tilde sign above the quantities in eqn. (A.6) and (A.9) will be dropped for a simpler notation.

### A.2 Used vector identities

\[
\nabla \cdot (k \times V) = V \cdot (\nabla \times k) - k \cdot (\nabla \times V) \tag{A.10}
\]

\[
(V \cdot \nabla) V = \nabla \left( \frac{V \cdot V}{2} \right) + \zeta k \times V \tag{A.11}
\]
Appendix B

Model Settings

B.1 Model Setup

B.1.1 WPS settings

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Table B.1: Excerpt of WPS variable settings.
APPENDIX B. MODEL SETTINGS

B.1.2 Namelist Settings

B.2 Model Level Heights

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Table B.3: Height of the vertical levels in the WRF domain over water surfaces (without terrain).
### B.2. MODEL LEVEL HEIGHTS

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<th>Variable Name</th>
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</table>

Table B.2: Excerpt of namelist.input variable settings.
Appendix C

Source Codes

C.1 NCL Script Vorticity Budget

Listing C.1: Vorticity Budget

```ncl
maxtime = dimsizes(file)
print(maxtime)
do ii = 1, maxtime(0) - 2 ; Loop over the files
ftb = addfile(files(ii - 1), "r") ; before
ftn = addfile(files(ii), "r") ; now
fta = addfile(files(ii + 1), "r") ; after

; Constants
R = 287
nr_smooth_iterations = 100

; Read z, lat, lon, f, terrain height from file
z = wrf_user_getvar(ftn, "z", 0)
l = wrf_user_getvar(ftn, "l", 0)
l = wrf_user_getvar(ftn, "l", 0)
f = ftn - F(0, :, :)
terrain = ftn - HGT(0, :, :)
ter = cut(terrain)

; Read data from t-1
utb = wrf_user_getvar(ftb, "ua", 0)
vtb = wrf_user_getvar(ftb, "va", 0)
wtb = wrf_user_getvar(ftb, "wa", 0)

; Read data from t:
utn = wrf_user_getvar(ftn, "ua", 0)
vtm = wrf_user_getvar(ftn, "va", 0)
wtm = wrf_user_getvar(ftn, "wa", 0)
ptn = wrf_user_getvar(ftn, "p", 0)
```

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C.1. NCL SCRIPT VORTICITY BUDGET

```ncl
; read data from t+1
utn=wrf_user_getvar(ftn,"tk",0)
uta=wrf_user_getvar(fta,"ua",0)
cta=wrf_user_getvar(fta,"ca",0)
hta=wrf_user_getvar(fta,"ha",0)

; Interpolate data on 3 Levels t-1
h1=100
h2=600
h3=1100
utblvl1=wrf_interp_3d_z(utb,z,h1)
utblvl2=wrf_interp_3d_z(utb,z,h2)
utblvl3=wrf_interp_3d_z(utb,z,h3)
vtblvl1=wrf_interp_3d_z(vtb,z,h1)
vtblvl2=wrf_interp_3d_z(vtb,z,h2)
vtblvl3=wrf_interp_3d_z(vtb,z,h3)

; smoothing
wrf_smooth_2d(utblvl1,nr_smooth_iterations)
wrf_smooth_2d(utblvl2,nr_smooth_iterations)
wrf_smooth_2d(utblvl3,nr_smooth_iterations)

; Interpolate data on 3 Levels t
utnlvl1=wrf_interp_3d_z(utn,z,h1)
unvlvl2=wrf_interp_3d_z(utn,z,h2)
unvlvl3=wrf_interp_3d_z(utn,z,h3)

; smoothing
wrf_smooth_2d(unvlvl1,nr_smooth_iterations)
wrf_smooth_2d(unvlvl2,nr_smooth_iterations)
wrf_smooth_2d(unvlvl3,nr_smooth_iterations)
```

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APPENDIX C. SOURCE CODES

```plaintext
wrf_smooth_2d(wtnlvl1, nr_smooth_iterations)
wrf_smooth_2d(wtnlvl2, nr_smooth_iterations)
wrf_smooth_2d(wtnlvl3, nr_smooth_iterations)
wrf_smooth_2d(ptnlvl2, nr_smooth_iterations)
wrf_smooth_2d(ttnlvl2, nr_smooth_iterations)

; Interpolate data on 3 Levels t+1
utalvl1 = wrf_interp_3d_z(uta, z, h1)
vtalvl1 = wrf_interp_3d_z(vta, z, h1)

wrf_smooth_2d(utalvl1, nr_smooth_iterations)
wrf_smooth_2d(vtalvl1, nr_smooth_iterations)

; Calculate density
rho_tnvl2 = ptnlvl2 / Ttnlvl2
print(rho_tnvl2(100, 100))

; Calculate vertical Vorticity Tendency for level 2 in the middle
votblvl2 = uv2vr_cfd(utblvl2, vtblvl2, lat(:,0), lon(0, :), 0);
votnlvl2 = uv2vr_cfd(utnvl2, vtnlvl2, lat(:,0), lon(0, :), 0)

votalvl2 = wrf_interp_3d_z(utalvl2, vtalvl2, lat(:,0), lon(0, :), 0)

dvodt = (votalvl2 - votblvl2) / (30 * 60) ; 30 min

; Calculate Vorticity on other levels for vertical gradient
votnlvl3 = uv2vr_cfd(utnvl3, vtnlvl3, lat(:,0), lon(0, :), 0)
votnlvl1 = uv2vr_cfd(utnlvl1, vtnlvl1, lat(:,0), lon(0, :), 0)

dvodt = (votalvl2 - votnlvl2) / (h3 - h1) ; 30 min

; Calculate Advection Term
advection_term = cut(utnlvl2) * dx_centered(votnlvl2) +
- cut(vtnlvl2) * dy_centered(votnlvl2) +
- cut(wtnlvl2) * (cut(vtnlvl3) - cut(vtnlvl1)) / (h3 - h1)

; Calculate Divergence Term
divergence_term = -(zeta + f) * (du/dx + dv/dy)

divergence_term = cut(votnlvl2 + f) * (dx_centered(utnlvl2) +
+ dy_centered(vtnlvl2))

; Calculate Tilting Term
tilting_term = -(dx_centered(wtnlvl2) * (cut(vtnlvl3 - vtnlvl1) / (h3 - h1)) +
- dy_centered(wtnlvl2) * (cut(utnvl3 - utnvl1) / (h3 - h1)))
```

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C.2 NCL Script SST Change

Listing C.2: SST Change

```ncl
ncdf = addfile("wrfinput_d01.nc", "w")

; output variables directly
SST = ncdf->SST
TSK = ncdf->TSK
LSM = ncdf->LANDMASK

ncdf->SST = SST+1 ;K
ncdf->TSK = where(LSM.eq.0, TSK+1, TSK)

; change SST and TSK by +1 K where LANDMASK is 0 (water surface)
```

The function `cut(...)` brings scalar fields to the same dimension.
Curriculum Vitae

Personal Data
Name: Johannes Sachsperger
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Address: Hinterhölzlgasse 5, 4100 Ottensheim, Austria

Education
1997 - 2001 Elementary school
2001 - 2006 Technical versatile education

Studies
Since 2007 Study of Meteorology at the University of Vienna, Austria
Since 2010 Research Assistant at the University of Vienna, Austria

Scientific Contributions
2011 Poster *The Atmospheric Wake of Madeira Island: the i-Wake Campaign* at European Geosciences Union Annual Meeting, Vienna, Austria
2011 Oral presentation *Die atmosphärische Wirbelstraße von Madeira* at Österreichischer Meteorologentag in Klagenfurt, Austria
2012 Poster *The Atmospheric Wake of Madeira Island* at Ocean Sciences Meeting, Salt Lake City, Utah
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