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The semantics of degree questions & Theories on negative island obviation effects

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To all the lambdas
that didn’t make it into this thesis.
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Introduction

The present thesis is concerned with semantic approaches to an account for negative island effects such as the contrast in (1)

(1.1) a. How much do you weigh?
      b. * How much do you not weigh?

(1.1b) is judged as clearly ungrammatical, yet why this should be the case is far from obvious. Similar construction with other kinds of interrogative phrases do not show the same pattern:

(1.2) a. Whom did you invite to the party?
      b. Whom didn't you invite to the party?

Here, the question in (1.2b) is unequivocally judged as grammatical. Except for the type of the element questioned, these two utterances seem to be identical in their syntactic structure, and there seems to be no straightforward way of explaining why one of the sentences is perfectly acceptably, while the other is, in essence, gibberish.

It is commonly assumed that these kind of ungrammaticalities arise due to violations of “island constraints” in syntax, that is, it is assumed a sentence like (1.1b) is ungram-
matical because the wh-phrase “How much” cannot be extracted out of its base position and moved to a higher position because negation blocks this kind of movement. Explaining the contrast between (1.1b) and (1.2b) isn’t just as simple, but accounts of this phenomenon have been brought forward for instance by Rizzi (1990), which will be discussed later.

1.1 Negative islands as weak islands

Note that the examples (1.1)-(1.2) contrast sharply with other island such as islands that arise due to Complex NP Constraint violations, e.g.

(1.3) a. * Whom did John invite a friend who knew?
   b. * How much did John invite a friend who weighed?

These islands differ from their counterpart in (1.1) in that they don’t allow any wh-phrase to be extracted, whereas in (1.1) and (1.2) only certain wh-phrases are blocked from movement. The former islands are thus commonly known as weak or, following Rullmann (1995), selective islands, whereas islands that block extraction of any kind of wh-element are known as strong islands.

Similar contrasts to the one in (1.1)-(1.2) have been observed for other types of wh-phrases, and in contexts other than negation. For instance, questions about manners show the same contrast:

(1.4) a. How did John behave?
   b. * How didn’t John behave?

And in addition to negation, a number of other contexts create the same restrictions on wh-movement (examples following Abrusán 2007, Rullmann 1995):

(1.5) a. Who did John regret that he invited?
   b. * How did John regret that he behaved?
   c. * How much does John regret that he weighs?
(1.6) a. Who did John deny that he invited?
   b. * How did John deny that he behaved?
   c. * How much has John denied that he weighs?

(1.7) a. ? Who do you wonder whether John invited?
   b. * How do you wonder whether John behaved?
   c. * How much do you wonder whether John weighs?

While these kinds of islands - following Abrusán (2007): presuppositional (examples 1.5, 1.6) and wh-islands (example 1.7) - pose interesting questions as well, I will not discuss them here, nor will manner questions be addressed in any detail. Cf., for instance, Abrusán (2007) or Fox (2007) to see how a semantic account of negative islands presented in this thesis could be extended to these cases as well.

Lastly, also note that the contrasts outlined in this section extend to other degree constructions as well, e.g. comparatives:

(1.8) a. John weighs more than Bill does.
   b. * John weighs more than Bill doesn't.

While I will touch on these constructions as well from time to time, a full investigation of this topic and the relevant literature would be beyond the reach of this thesis.

1.2 Syntactic accounts of negative islands

There are a number of publications that propose an entirely syntactic account of the facts outlined above, going back to Ross (1984) who first discovered negative islands. Among them are, for instance, Rizzi (1990), Cinque (1990), De Swart (1992), Manzini (1992, 1999) and Beck (1996).

Most of these proposals are building on the notion that negation blocks extraction of certain phrases, among them degree expressions, due to certain locality requirements. They differ in how exactly they implement this idea, and in particular in how they explain why only some phrases are affected by weak islands.
Szabolcsi and Zwarts (1990, 1993) propose a somewhat different account based on the algebraic structure of the domain of degrees. In their proposal negative islands arise due to the corresponding operation - taking complement - not always being defined on that domain.

However, syntactic accounts of negative islands are not the focus of this thesis and as such I will not go into them in any detail.

Furthermore, a key argument against an approach based on syntactic constraints has been presented in Fox & Hackl (2006), who note that certain modal constructions can obviate negative islands, as in (1.9).

\[(1.9) \quad \begin{align*}
&\text{a. } * \text{ How fast didn't you drive?} \\
&\text{b. } \text{How fast are you not allowed to drive?} \\
&\text{c. } \text{How fast are you required not to drive?}
\end{align*}
\]  
(from Abrusán & Spector 2008, in progress)

This is rather unexpected under a syntactic account for negative island constraints. If the negator “not” is an intervener that blocks wh-movement then it is puzzling that the presence of a modal operator should suddenly allow the wh-phrase to escape. Further testimony to the difficulty of explaining these facts is that, to the best of my knowledge, no syntactic account for the contrast in (1.9) has been proposed to date.

### 1.3 Outline and motivation

A number of proposals have thus been brought forward that focus on the semantics of the constructions involved and fare better in explaining the data in (1.9). In particular, this thesis will look at the theories presented in Fox & Hackl (2006) and Abrusán & Spector (2008, in progress). A common thread in all of these theories will be the idea that, roughly:

- (i) all questions presuppose that they have a (unique) maximally informative answer, that is, a single answer that is true and entails all other true answers,
- (ii) due to the semantics of degree constructions this presupposition will never be met in some cases, i.e. is a contradiction, and
(iii) certain kinds of contradictions, including the aforementioned, can lead to ungrammaticality.

Claim (i) essentially goes back to Dayal (1996) and is shared, without major differences in implementation, by both Fox & Hackl (2006) and Abrusán & Spector (2008, in progress). This will be presented in more detail as part of the next chapter, in the broader context of the semantics of questions in general.

Claim (ii) is where Fox & Hackl (2006) and Abrusán & Spector (2008, in progress) diverge the most. The former posit a semantic framework that is based around the idea that the scales degrees refer to are always dense, and that for this reason in some cases there cannot be a most informative answer. The latter pursue a radically different approach, using a semantic system based on intervals of degrees; Here the relevant factor will be the requirement that there be a most informative interval of degrees. This discussion will be the focus of a large part of this thesis.

Finally, claim (iii), building on Gajewski (2002), is shared by both approaches, although with subtle differences in implementation. While some of what Fox & Hackl (2006) claim is also entailed by Abrusán & Spector (2008, in progress)’s approach, the differences have far reaching consequences for our understanding of how the linguistic system works.

Fox & Hackl’s approach in particular makes strong claims about a presumed Deductive System within the linguistic facility, and this system being blind to lexical information. This would, then, mean that lexical information only comes into play quite “late” in the process of decoding of linguistic information. Abrusán & Spector’s approach, on the other hand, makes fewer assumptions about the structure of the linguistic facility. However, the idea that certain, but not all, contradictions in meaning can lead to unacceptability of utterances is central to this approach as well.

These questions, in my opinion, are more fundamental in nature than most other current debates in linguistics and it is these aspects, thus, that make the theories I will discuss relevant beyond the topic at hand. I would even assume that a discussion of these issues will likely be appealing to an audience wider than that interested in formal semantics per se. It will be one goal of my thesis to establish the relevance of the cur-
rent debate to a broad linguistic audience and to highlight the implicatures the current debate has on our conception of the structure of the linguistic system. I will return to this issue at the end of my thesis.

The structure of the remainder of this thesis is as follows:

Chapter 2 will present some standard approaches to the semantics of questions in general, e.g. Hamblin (1958, 1973), Karttunen (1977), Groenendijk & Stokhof (1982) and Heim (1994). An overview over the most important issues in this field will be provided, e.g. exhaustivity, embedded questions. A particular focus will be on the relationship between domains over which wh-phrases quantify, sets of propositions and sets of possible worlds, and how these play together in the semantics of questions.

Chapter 3 will then turn to the semantics of degree constructions. Competing approaches such as Rullmann (1995), Schwarzschild & Wilkinson (2002) and Beck & Rullmann (1996, 1999) will be compared.

Chapter 4 will focus on negative islands in degree questions and the theories presented in Fox & Hackl (2006) and Abrusán & Spector (2008, in progress). Rullmann's (1995) account of negative islands will be briefly outlined.

Chapter 5 will turn to differences between the two theories presented in chapter 4, and their limitations when it comes to modalized degree predicates. A unified account will briefly be considered, before an approach based on an alternative Generalized Quantifier reading for degree questions will be outlined, as proposed by Fox (2010), in turn based in Spector (2007, 2008).

Chapter 6 will briefly outline how the issues presented in this thesis are related to our understanding of how syntax and semantics interact.

As a final remark, one of the main objectives of this thesis is to make the discussion at hand accessible to a broad linguistic audience. It appears to me that the field of natural language semantics is met with reservation (if not sheer angst) by many students, the reasons for which I can only speculate about. With this in mind, it is an explicit aim of
this thesis to make one of the current debates in this field accessible to those without a strong background in formal linguistics. As a consequence, I have decided to present the ideas involved in a rather informal way, while still trying to keep true to their spirit. If anywhere I abstracted away from the technical details too much I apologize to both the reader and the original authors.
Approaches to the semantics of questions

2.1 Some initial remarks on the semantics of natural language

Before going into the semantics of interrogatives, let me remind the reader of some basic concepts in the field of natural language semantics, following Heim & Kratzer (1998). The basic idea behind most research in this area, going back as far as Frege, is that the meaning of a sentence like (2.1) can be computed from the meaning of its parts.

(2.1) There is a book on my desk.

It may seem trivial to note that the sentence in (2.1) is true if and only if it is in fact true that there is a book on my desk. Yet what is less obvious is how we are able to derive the meaning of an infinite number of complex sentences, only by virtue of the
meaning of their parts. I will not go into the mechanics of compositional semantics in any detail here, however confer Heim & Kratzer (1998) for a comprehensive introduction.

I will point out however, that a fundamental idea in the field of semantics is that it is possible to assign *truth-conditions* to propositions, that is, informally speaking, to list, in detail, what needs to be a fact of the world for a sentence to be true. Considering (2.1), this would be something along the lines of (2.2).

(2.2) i. There is a book, x.
ii. There is a table, y.
iii. y is owned by the speaker of ō.
iv. x is in a particular spatial relation, “on”, with y.

Again, listing these conditions is trivial in this example. Note, however, that once one entertains a notion of truth-conditions of a statement, it is easy to see that any one of these conditions could be false as well; that is, it is easy to arrive at the observation that there is an infinite number of possible states of affairs, an infinite number of possibilities as to what may be true or, in other words, an infinite number of *possible worlds*, only in some of which any given statement (tautologies aside) will be true. It is thus a central observation in formal semantics that there is a direct relationship between a proposition and the set of possible worlds in which it is true.

(2.3) A proposition is equivalent to a set of possible worlds.

While this equivalence in itself doesn’t add much to our discussion at this point, it is worth remembering that these two concepts – a proposition & its truth-conditions, and a set of possible worlds – are but two sides of the same coin. While some of the theories presented later will tend to focus more on one, some will use the two concepts interchangeably, and keeping in mind both ways of looking at any given semantic

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1 ōii and ōiii are actually presuppositions.

2 in accordance with established terminology, I will stick to the latter.

3 or, equivalently, its complement, the set of worlds in which a give proposition is *false*; however, sticking with what is established standard (and yields simpler formulas), I mention this only as a side note.
problem will allow for an easier understanding of the differences between those theories.

Notice also that the domain of proposition isn’t an entirely unstructured field; there is a number of relations in which two propositions can stand to each other, e.g. they can be contradicting each other, they can be equivalent, or one can entail the other. It is essential to realize that all these can be formulated in terms of sets of possible worlds as well. For instance, if a proposition P entails another proposition Q, it means that whenever (i.e. in all the possible worlds in which) P is true, Q is also true, or in other words, the set of worlds in which P is true is a subset of the worlds in which Q is true, cf. (2.4).

\[(2.4)\]
\[
\begin{align*}
\text{i. } & \text{“P”} \rightarrow \text{“Q”} \\
\text{ii. } & P \subseteq Q
\end{align*}
\]

\[(2.5)\]
\[
\begin{align*}
\text{i. } & \text{Bill came.} \\
\text{ii. } & \text{John came.} \\
\text{iii. } & \text{Bill and John came.}
\end{align*}
\]

Lastly, let me point out that in cases like (2.5) there is also an obvious connection between the relation “entails” on the level of propositions, and the relation “subset of” on the domain of sets of individuals.

\[(2.6)\]
\[
\begin{align*}
\text{i. } & \text{“Bill and John came.”} \rightarrow \text{“Bill came.”} \\
\text{ii. } & \{\text{Bill}\} \subseteq \{\text{Bill, John}\}
\end{align*}
\]

Noting this effect, called distributivity, might again seem trivial in this simple case, but will get considerably more complicated once we consider not individuals and sets of individuals, but degrees and sets of degrees.

### 2.2 Hamblin and Karttunen

Turning to questions, it it far less obvious how to treat them semantically; As should be plain obvious, questions certainly have meaning, but they do not have truth-values,
they aren’t “true” or “false” in any sense. Consequently, it seems difficult to capture their meaning within the framework outlined in the preceding section, which is focused on computing the truth-value and truth-conditions of an utterance via the meaning of its constituents. However, as we will see, answers do have truth-values, and all of the approaches presented in the following sections will link questions to their answers in one way or another. In this, all of them are similar, although some of them will be close to syntax, while others will be more abstract and look at propositions in terms of sets of possible worlds. Still, there is no fundamental difference between these approaches as long as the equivalence of propositions and sets of possible worlds is kept in mind.

While the most relevant theories in the field of the semantics of questions will be presented, a number of others will be left out. For a good overview of recent works in this field and some open questions, cf. for instance Hagstrom (2003).

Hamblin (1958)

The first paper to discuss the semantics of interrogatives was Hamblin (1958). In it, Hamblin proposes the following three postulates:

1. An answer to a question is a statement.
2. Knowing what counts as an answer is equivalent to knowing the question.
3. The possible answers to a question are an exhaustive set of mutually exclusive possibilities.

(Hamblin 1958)

Postulate 1 states that while an answer such as “John” might commonly be an acceptable answer to the question “Who came?”, it is merely the elided form of the full answer “John came.”

Postulate 2 presents the idea that a question is equivalent to the set of all propositions that are possible answers to it, i.e. a set of sets of possible worlds. This concept, with various modifications, will be central to a number of other proposals as well. In how

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4 although Karttunen (1977:4), considering questions equivalent to the corresponding “I ask you (to tell me) …” utterances, argues that questions do have a truth-value, and that the pointlessness of asking for it stems from the fact that they are true by virtue of being asked.
exactly this idea is implemented, the various theories differ, and it is far from clear what should count as a possible answer to a question.

Postulate 3 further claims that the set of answers essentially is a partition of all possible worlds, i.e. each possible world is included in the extension of one, and only one, of the possible answers. Or in other words, in any given possible state of affairs, one and only one of these answers will be true.

**Hamblin (1973)**

Hamblin (1973) differs from his earlier account in two central aspects.

Firstly, Hamblin moved to an approach closer to syntax, and focused on determining the meaning of a question compositionally, similarly to the approach outlined in section 2.1 of this thesis. The central assumption is that a wh-phrase like “Who” or “What” is structurally similar to the corresponding phrase in a declarative sentence, e.g. in “Who came?” and “John came.”, “Who” and “John” seem to be of a similar nature. The meaning of a question then is the set of all propositions that one gets when replacing the wh-phrase with all the relevant options, e.g. people in the case of “Who”, thus restricting the set originally put forward in his original proposal. For instance, the denotation of the question (2.7a) would be the set in (2.7b).

\[
\begin{align*}
(2.7) & \quad \text{a.} \quad \text{“Who came?”} \\
& \quad \text{b.} \quad \{\text{“John came”, “Bill came”, “Fred came”,…}\}
\end{align*}
\]

Secondly, this move implicitly discards his original third proposal, as in this approach answers need not be mutually exclusive anymore. As should be obvious in the above example, there could conceivable be worlds where both John and Bill came.

**Karttunen (1977)**

Building on Hamblin’s (1958, 1973) earlier works, Karttunen (1977) proposes a framework that captures both matrix questions as well as embedded questions. He follows Hamblin (1973) in that the meaning of questions is built compositionally, but moves to include only *true* answers in the question denotation, noticing that question-
embedding verbs like “know” behave as if they embedded all the corresponding true answers, e.g. if only John and Bill came, (2.8a) amounts to (2.8b).

(2.8) a. I know who came.
    b. I know that Bill came, I know that John came.

As Karttunen points out, in this framework the denotation of a question is the set of propositions whose conjunction constitutes the true and complete answer to the question.

2.3 Weak and strong exhaustivity

As Groenendijk & Stokhof (1982, 1984, 1990) point out, while Karttunen’s (1977) analysis of the semantics of questions succeeded in relating the meaning of matrix questions to that of embedded questions, there are some cases where his theory predicts weaker readings than are generally agreed on by speakers. Consider for instance (2.9).

(2.9) John knows who came to the party.

Imagine a case where, say, Bill, Mary, Sue and Fred where thought to possibly come, and only Bill and Mary came. In this case, if John knows that Bill came, and John knows that Mary came, yet knows nothing else, i.e. doesn’t know if either Sue or Fred came, Karttunen’s approach would predict 0 to be true, despite the fact that it is commonly judged as false in such a situation.

Groenendijk & Stokhof’s partition semantics

To account for this, as they label it, “weakly exhaustive” reading predicted under Karttunen (1977), Groenendijk & Stokhof (1982, 1984, 1990) propose a rather different approach. Returning to Hamblin’s original postulates, they analyze questions as sets of mutually exclusive answers. In their proposal, the focus is on propositions as sets of possible worlds, and sets of answers as disjoint sets of possible worlds. In other words, while propositions are sets of worlds, questions are sets of propositions, i.e. sets of sets of possible worlds. As these sets of possible worlds are, going back to Hamblin’s (1958) original proposal, mutually exclusive and exhaustive, we arrive at a partition of (all the) possible worlds, i.e. a question divides up the set of all possible worlds into compart-
ments, such that each possible world is in exactly one of them. For instance, assuming that only Bill and John are relevant, the question in (2.10a) would induce the four cells described in (2.10b) - (e).

(2.10) a. “Who came?”
   b. “John and Bill came.”
   c. “John came and Bill didn’t.”
   d. “Bill came and John didn’t.”
   e. “Neither/Nobody came.”

Knowing the answer to a question then means knowing in which of these compartments our actual world is in, and asking a question is asking for the same information.

This gives us another reading of embedded questions such as (2.9), which Groenendijk & Stokhof call the strongly exhaustive reading. Under this reading a sentence like (2.9) is only predicted to be true if one knows who came, as well as who didn’t.

Another advantage of Groenendijk & Stokhof’s theory is that it allows to define not only what constitutes a complete answer to a question, but also what counts as a partial answer. For instance, answering “John didn’t come” to (2.10a), one doesn’t convey all the information that was asked for, or in Groenendijk & Stokhof’s terms, one doesn’t pinpoint in which compartment (2.10b) - (e) the actual world is located. One does, however, rule out (2.10b) and (c), and thus conveys some of the information that was asked for, narrowing down the search so to speak.

**Heim (1994)**

However, as pointed out by Heim (1994), not all question-embedding verbs make use of such a strong notion of exhaustivity. Consider for instance the contrast in (2.11).

(2.11) a. It surprised me who showed up.
   b. It surprised me who didn’t show up.

It seems (as discussed by Heim 1994:139, also Berman 1991) that these two sentences have a subtly different meaning, namely that when one is surprised who came it seems one is surprised just that for every person who came, that they came, but not necessar-
ily that those who didn’t come, didn’t. This would not be predicted by Groenendijk & Stokhof, and it seems difficult to derive such a meaning from their theory, as Heim argues. It seems, then, that at least some question-embedding verbs make use of a notion of answer closer to that of Karttunen (1977), and that somewhere in the process of computing the meaning of a question such a notion should be in play.

Heim thus shows that it is indeed possible to derive Groenendijk & Stokhof’s meaning for questions from Karttunen’s set of true answers. She does that by pointing out that Karttunen expressedly acknowledged the weakly exhaustive reading predicted by his account, except in one case: Whenever an embedded question denotes the empty set, knowing it should require knowing that it denotes the empty set.

Consider first what would be predicted without this additional requirement. Under Karttunen’s account, to know who called is equivalent to knowing every true answer to the question “Who called?”. So if Bill and only Bill called, knowing who called is knowing that Bill called. If nobody called, then knowing every true answer to the question “Who called?” amounts to knowing every proposition out of an empty set of proposition - in other words, knowing nothing (of relevance) at all.

With Karttunen’s added requirement however one is required to know that the complete, true answer to “Who called?” is the empty set, i.e. one is required to know that nobody called. Heim points out that if this requirement is extended to all cases, the result is equivalent to Groenendijk & Stokhof’s strongly exhaustive reading:

If knowing a question is knowing not only the complete, true answer, but also that this knowledge is the complete, true answer, one obviously knows that all other possible answer are in fact not true, or not complete.

Thus, Heim defines two notions of “answer”, answer$_1$ and answer$_2$:

\[(2.12) \quad \text{a. } \text{answer}_1 \text{ is the set of all true answers, similarly as in Karttunen (1977).} \]
\[\quad \text{b. } \text{answer}_2 \text{ is answer}_1 \text{ and knowing that this is answer}_1.\]

---

5 Heim actually points out that it is possible to construct cases where the two accounts aren't equivalent and her account gives wrong predictions. However, as Heim also suggests a way of addressing this issue with some minor adjustments to her theory, I will leave this aside as a technicality.
In Heim’s framework, the basic denotation of a question is answer_1, while answer_2 is derived from answer_1. This way, strong exhaustivity can be accounted for by stipulating that some question-embedding verbs make use of answer_2, while situations in which answer_1 is needed can still be accommodated.\(^6\)

However, as Heim points out, while answer_2 can be derived from answer_1, the reverse is not possible. Since there are question-embedding verbs that arguably require answer_1, such as “surprise”, this is an argument in favor of an analysis that includes this notion of “answer” at some stage in the derivation of the meaning of a question.

One other thing that Heim adds to Karttunen’s (1977) original theory is the use of plural semantics for the domains of individuals the wh-phrases so far considered range over, i.e. in her theory the answer_1 to (2.13a) is not (2.13b), but (2.13c).

\[(2.13)\] Assuming that only John, Bill and Fred came:

a. “Who came?”

b. (“John came”, “Bill came”, “Fred came”)

c. (“John came”, “Bill came”, “Fred came”, “John and Bill came”, “John and Fred came”, “Bill and Fred came”, “John and Bill and Fred came.”)

It is worth pointing out (although Heim doesn’t explicitly mention this; but see Dayal 1996 and the following section of this thesis) that once one considers plural individuals when computing answer_1 the requirement that the elements of answer_1 jointly constitute the complete, true answer is, in the case of questions about plural individuals, in a sense redundant: In the above example, for instance, the element “John and Bill and Fred came.” is, by itself, already the complete (and true) answer to the question. It is, then, not necessary to allow for a conjunction of statements to be the complete true answer: By virtue of distributivity among plural individuals\(^7\) taking a statement to apply to the sum of two individuals (or sets of individuals) is equivalent to taking the conjunction of the statements about each of the individuals (or sets of individuals).

\(^6\) Heim’s approach actually solves another problem Karttunen’s approach had compared to Groenendijk & Stokhof’s, that of de re versus de dicto readings of some questions. As this is tangential at best to this discussion I will not go into this in any detail. The reader is referred to Heim’s (1994) paper, or to Hagström (2003) for a discussion of this topic (and others).

\(^7\) That is, leaving aside non-distributive predicates such as “meet” for the moment.
2.4 Maximal informativity

A similar point was made by Dayal (1996) who argued that a question was, in fact, distinct from its answer, the former being equivalent to the set of possible answers, in accordance with Hamblin’s (1958) original proposal, while the latter is identified with the maximally informative true answer, that is, the single true answer that entails all other true answers. In this framework, the denotation of the question in (2.14) would be the set in (2.14), while its answer would be “John and Bill and Fred came”. I will from now on refer to this set of possible answers (including those based on plural individuals) as the H/K (i.e. Hamblin/Karttunen) set or denotation of a question.

Dayal also made the point that a question presupposes that such a unique maximally informative (true) answer exists. To exemplify this, consider (2.14) in a context where both Bill and John are liked by Sue:

(2.14) Which boy does Sue like?

In this context, this question sounds odd, and under Dayal’s proposal this is easily explained: The singular “Which boy” ranges only over singular individuals, not groups of them, and consequently the set of possible answers would contain both “Sue likes Bill” and “Sue likes John”, but not “Sue likes Bill and John”. Since neither of the former two entails the other (but both are true in the context), the presupposition that there be a single maximally informative true answer is not met.

Realizing, then, that the set of true answers, as a subset of the set of possible answers, has a unique maximally informative element, one can then define an operator that picks out this element (as do, for instance, Fox 2007 or Fox & Hackl 2006). The operator Max$_{\text{inf}}$, defined in Fox & Hackl (2006) and amended in Fox (2007), picks out the unique most informative true element of a set of propositions, if such an element exists. Notice that applying Max$_{\text{inf}}$ to the H/K set of a question only gives us answer$_1$, not answer$_2$. To get answer$_2$ one still needs to stipulate that this maximally informative

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8 A similar proposal was also made by Jacobson (1995).

9 Fox (2007) and Fox & Hackl (2006) define this operator for any set of propositions, so long as a unique maximally informative true element exists. As they point out, Max$_{\text{inf}}$ can also be used in a number of constructions other than questions.
answer is in fact just that, the *maximally* informative true answer, i.e. that there is no other true answer that isn’t entailed by it.

Fox (2007), Fox & Hackl (2006), among others, also relate to problem of exhaustiveness in questions to other problems, for instance scalar implicatures or the grammaticality of the particle “only”. Notice for instance that saying that “John and Bill came.” is the most informative true answer to a question is akin to saying that only John and Bill came. And in most contexts it is assumed that from (2.15a) follows (2.15b), i.e. claiming (2.15a) implicitly also conveys the information that (among a set of alternative claims) (2.15a) is the strongest (i.e. most informative) claim the speaker believes.

(2.15) a. John is required to read (at least) four books.
 b. John is not required to read more than four books / at least five books.

Scalar implicatures are not the main focus of this thesis, but I will briefly mention them when discussing Fox & Hackl (2006).

### 2.5 Conclusion

Starting with early approaches such as Hamblin (1958, 1973) and Karttunen (1977), various approaches to the semantics of questions have been presented. As a basis for the explications in later chapters, the following will be assumed, based on the theories hitherto presented:

The basic meaning of a question is its H/K denotation, that is the set of all possible answers. A possible answer, here, is roughly speaking the statement one gets by substituting an element from the domain over which a wh-phrase ranges for that wh-phrase. In case of singular wh-phrases this includes only atomic individuals, while in the case of plural wh-phrases any sets of individuals from the domain of quantification are taken into account to build the H/K set. Notice that it is not clear from these stipulations what the H/K set of a question about other domains than individuals, in particular a degree question, will be. One of the key differences between the approaches to those questions will be what kind of plural entities will be admissible to the H/K denotation of these questions.
Whenever this set has a unique maximally informative true element, that is a single true proposition that entails all other true propositions in the H/K denotation of a question, it is feasible to speak of the answer to a question. Notice that it is by no means necessary that this be true. However, as Dayal (1996) pointed out, any question crucially presupposes that such a unique element exists. Whether it does exist depends on the structure of the H/K set of a question, and thus, in turn, on the structure of the underlying domain over which the wh-phrase(s) in a question range.

Following Fox (2007), among others, it is possible to define an operator, Max_{\text{inf}} on this (or any other) set of propositions that picks out the most informative true element of this set, if such an element exists. This concept of maximality of informativeness can then be used in defining the notion of strong exhaustivity in questions, as well as in explaining other related phenomena, such as scalar implicatures or the meaning of the particle “only”. Maximal informativity will be a key concept in the proposals presented in chapters 4 & 5.

Lastly, it is possible, though this is not central to the arguments to follow, to formulate the above ideas in terms of sets of possible worlds and Groenendijk & Stokhof’s framework as well. Roughly speaking, the partition of possible world induced by a question could be defined by the set that one gets as a result of applying an exhaustivity operator to each member of the H/K denotation of a question: That is, while the H/K denotation of a question is the set of all possible (weakly exhaustive) answers to a question, the denotation of a question as proposed by Groenendijk & Stokhof is the set of all possible strongly exhaustive answers to a question. Dayal’s (1996) claim that a question presupposes the existence of a unique most informative true answer then translates to the presupposition that (i) it is possible to exhaustify each member of the H/K denotation of the question, and that (ii) the resulting set of propositions is a proper partition of all possible worlds.\textsuperscript{10}

\textsuperscript{10} In the case of singular wh-phrases such as “Which boy came?” this would then translate to the presupposition that exactly one boy (or none at all) came. Exhaustifying each proposition of the form “x came”, x being a single boy, results in a partition of exactly those possible worlds where exactly one boy came.
Degree constructions

While the denotation of simple questions about individuals was introduced in chapter 2, this chapter will now focus on the meaning of degree questions, and will touch on other degree constructions such as comparatives as well.

Following recent literature (cf. for instance von Stechow 1984, Rullmann 1995, Abrusán 2007 and others) in this chapter as well as the remainder of this thesis it will be assumed that the meaning of a degree construction such as “x feet tall” corresponds in some way to a scale, respectively point(s) on a scale, that is, degrees can be ordered by a relation such as “taller than”, “faster than”, etc. How exactly this is formally defined, for instance by making reference to equivalence classes of individuals, will only be mentioned as far as it is relevant to the arguments presented here. For a more detailed discussion of the ontology of degree cf. for instance section 2.10 of Rullmann (1995).

3.1 Issues faced by degree questions

Notice, first, that unlike (atomic) individuals, degrees such as one’s height, weight, or others, are naturally ordered by a “shorter than”, “lighter than”, or other, relation. This

or equivalently “taller than”, “heavier than”, etc.
gives rise to a richer field of constructions involving reference to degrees, such as, for instance, comparatives or equatives and differential comparatives, as exemplified in (3.1).

(3.1) a. John is taller than Bill is.\textsuperscript{12}
b. John is as tall as Bill is.
c. John is two inches taller than Bill is.

Providing a comprehensive treatment of the semantics of these construction would be far beyond the scope of this thesis, however they will be used at times to illustrate some of the points made in the literature presented.

Also, notice that it isn’t entirely clear what the meaning of a “plain” degree such as in (3.2) is. While it seems obvious that (3.2a) is meant to convey the information that John is exactly five feet tall, there is arguably contexts where it is judged as true even if John is in fact taller than five feet. Similarly, while (3.2b) does have a reading in which each boy has the same height, five feet, a more natural reading would be that (3.2b) means that each boy is at least five feet tall.

(3.2) a. John is five feet tall.
b. Every boy is five feet tall.

There seems to be no clear consensus in the literature as to whether the “exactly” or the “at least” reading is the basic denotation of a degree expression,\textsuperscript{13} however more recent works seem to favor the “at least” approach. Unless otherwise stated, the “at least” meaning will be assumed in this thesis.

Lastly, while in the case of singular and plural individuals it was rather straightforward to see how predicates combine with pluralities, in the case of degrees this is less obvious. In the case of simple statements like (3.3a) it seems clear that only singular de-

\textsuperscript{12} Following Rullmann (1995) and others, phrasal comparatives (and similar equatives, etc.) such as (i) will not be considered in this thesis. For a comprehensive list of references on the nature of phrasal comparatives cf. Rullmann (1995), chapter 2, footnote 4.

(i) John is taller than Bill.

\textsuperscript{13} Rullmann (1995) for instance uses an “exactly” meaning, while Beck & Rullmann (1999), Fox & Hackl (2006) and others use an “at least” reading.
degrees can be taken as an argument, hence the infelicity of (3.3b), but in the case of (3.3c) this is less obvious (and an “up to” reading seems the most natural in this case). (3.3d) seems to suggest that the speaker is uncertain as to what the speed limit is, but can hardly convey the information that it is allowed to drive at exactly 120 km/h, and to drive at exactly 130 km/h, but not at any other speed, as could be expected in analogy with (3.3c).

(3.3)  
   a. I am five feet tall.  
   b. # I am five and six feet tall.  
   c. You are allowed to drive 130 km/h here.  
   d. ? You are allowed to drive 120 or 130 km/h here.  
   e. You are allowed to take *Syntax 101* or *Phonology 101*.

The question as to what kind of pluralities of degrees can combine with predicates, and in particular which of them can be used in generating the H/K denotation of a question, will be central to some of the discussion in following chapters. At the very least, propositions about single degrees will have to go into the H/K denotation of questions, but what other sets of degrees will be considered will have to remain open for the time being.

It is clear at this point, however, that there are two independent relations with which degrees or sets of degrees can be compared: One is position on a scale, that is the relation “smaller than” etc. when talking about single degrees (or suitable sets of degrees, as we will see); The other is the relation “is a subset of” when talking about sets of degrees. Both can translate to relations of entailment on sets of propositions that take degrees as an argument, such as the H/K denotation of a question, yet how it is assumed that they do varies with the proposals that will be presented. For instance, if a plain “at least” meaning of degrees is assumed, it is easy to see that in any case where “being *x* feet tall” is true, “being *y* feet tall” would also be true for any *y* ≤ *x*.

Further, it is an obvious observation that, logically, there is a difference between “less than” - “<” - and “less than or equal” - “≤”. There is a variety of ways of expressing these relations in natural language, both for measurable adjectives, as well as for measurements of cardinality, as in (3.4) and (3.5).
(3.4)  
   a. John is at least / at most five feet tall.  
   b. John is more / less than five feet tall.  
   c. John is five feet tall or taller.  
   d. John is no taller than five feet.  
   e. John is between five and six feet tall.  

(3.5)  
   a. John read at least / at most five books.  
   b. John read more than five books.  
   c. John read five books or more.  
   d. John read no fewer than five books.  
   e. John read up to five books.  

It should be pointed out at this point that not all of these expression will be considered when generating the H/K set of a question, and a discussion of the semantics of all these construction would be beyond the scope of this thesis. However, for some interesting results on the semantics of “no more than”-expressions see Nouwen (2008).

3.2 Maximality in degree constructions

The idea that a kind of maximality is in play in some degree constructions was argued for, first, by von Stechow (1984), and then by Rullmann (1995) who focused mainly on negative islands in degree constructions.

In this framework, the meaning of a comparative such as (3.6a) is as described in (3.6b)\(^\text{14}\). A maximality operator $\text{max}$ is defined in a similar, but not quite equivalent, way as Fox’s (2007) $\text{Max}_{\text{inf}}$ operator, picking out the maximal degree in a set of degrees.

(3.6)  
   a. John is taller than Bill is.  
   b. John is taller than $\text{max}$ (any degree $d$, such that Bill is $d$-high).

The max-operator is obviously redundant in this simple example, but consider (3.7)\(^\text{15}\):

\(^\text{14}\) examples due to Rullmann (1995)
(3.7)  a.  John swam faster than Bill could run.
     b.  * John swam faster than at the unique speed at which Bill could run.
     c.  John swam faster than at the maximal speed at which Bill could run.
     (Rullmann 1995:54)

Clearly here the notion of maximality is needed to account for the correct reading. Further, as Rullmann points out, maximality can also explain certain ambiguities in “less than” constructions. Consider, for instance, (3.8), also due to Rullmann:

(3.8)  a.  The Helicopter was flying less high than a plane can fly.
     b.  “Because the helicopter was flying less high than a plane can fly, the jet fighter could easily fire at it from above.”
     c.  “The jet fighter was trying to chase the helicopter, but because the helicopter was flying less high than a plane can fly, the jet fighter crashed into a building.”
     (Rullmann 1995:84)

Clearly, (3.8a) has both a “less than maximum” reading, as exemplified in (3.8b), as well as a “less than minimum” reading, as in (3.8c). As Rullmann points out, although this ambiguity could be accounted for by stipulating a minimality operator to handle the less-than-min reading, a unified account is also possible, analyzing “higher” as “high” and the comparative morpheme “-er” (and observing that “less” is the comparative form of “little”, thus “little + -er”), the difference between the two readings then stems from two different underlying structures:

(3.9)  a.  The helicopter was flying -er little high than a plane can fly high.
     b.  The helicopter was flying -er little high than a plane can fly little high.
     (Rullmann 1995:93)

If (3.8a) is analyzed as being the elided form of (3.9a) it has the less-than-max reading, if it is analyzed as in (3.9b) it gets the less-than-min reading. This also explain rather straightforwardly why the equivalent sentence with “higher” doesn’t show this ambiguity.

In my opinion, further evidence for this comes from the fact that a sentence like (3.10) doesn’t show an ambiguity either, although this wasn’t noted by Rullmann:
(3.10) The helicopter wasn’t flying as high as a plane can fly.

Rullmann, however, notes that this account doesn’t extend to cases such as (3.11), and leaves this open for future research.

(3.11) The helicopter was flying lower than a plane can fly.

(Rullmann 1995:94)

Moreover, as Rullmann notes, this gives a feasible notion of exhaustivity in degree questions: A question like (3.12a) asks for the maximal degree to which John is tall, this clearly being the most informative answer.15

(3.12) a. How tall is Bill?
  b. “What is the maximal degree \( d \), such that Bill is \( d \)-tall?”

Rullmann goes further by relating this to the notion of exhaustivity in questions about (plural) individuals as well. As was already pointed out in section 2.1, there is an obvious connection between the “subset of” relation on sets of individuals and the “entails” relation on propositions that take them as an argument. As Rullmann points out, if John, Bill and Fred (and only they) came, then the maximal set of individuals who came is that consisting of John, Bill and Fred. And while all of the propositions in the H/K set of (3.13a), spelled out in (3.13b), would be true, it is the maximal element of this set (with regard to the “subset of” relation) that is the exhaustive answer to (3.13a).

(3.13) a. Who came?
  b. (“Bill came”, “John came”, “Fred came”, “John and Bill came”, “Bill and Fred came”, “John and Fred came”, “John and Bill and Fred came”)

Similarly to Heim (1994) he then defines strong exhaustivity, although relying on maximality on the domain of individuals (respectively domains), not maximality (with regard to entailment) on the set of true answers.

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15 Rullmann (1995) assumes an “exactly” meaning for plain degrees, in which case the maximality requirement in a simple case like (3.12a) is obviously redundant. It would, however, contribute in case of an “at least” reading, and would then in this particular case also give the desired result.
Rullmann makes a number of other interesting points, most of which I will have to leave aside though. However, a central point of his thesis is an account of negative islands, to which I will return in section 3.4.

For the moment let me conclude the discussion of Rullmann (1995) by pointing out that Rullmann's approach says little about the algebraic structure of degrees, in particular it doesn't address the subject of how plurals of degrees could be formed. The above cited example 0 obviously makes reference to both a minimum and a maximum degree of height at which a plan is able to fly, yet this isn't explicated. The next section will thus introduce one approach that focuses explicitly on sets of degrees.

Lastly, let me point out that I think that a (minor) theoretical shortcoming of Rullmann's (1995) proposal is his assumption of an “exactly” meaning for degrees. While in comparisons the use of a maximality operator is motivated independently, there is, a priori, no reason to stipulate a similar semantics for degree questions under this assumption. Under an “at least” reading on the other hand maximality would follow from maximal informativity, and not merely coincide with it (in simple cases).

3.3 Intervals of degrees

Schwarzschild & Wilkinson (2002) take a rather different route, noting that existing proposals cannot account for a number of phenomena, mainly involving quantifiers in the scope of a “than”-clause, for instance (example from ibid:9ff):

(3.14) Q is taller than everybody else is.

Given the situation as in Fig. 1, representing everybody’s height, (3.14) would obviously be judged false, yet as Schwarzschild & Wilkinson elaborate, the approach pursued by both von Stechow (1984) and Rullmann (1995) seems to give a wrong prediction: Under an “exactly” meaning, the set denoted by \{d: everybody else is d-tall\} is empty, and its maximum thus undefined. (3.14) is thus predicted to be infelicitous in

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16 They also elaborate on NPI-licensing, but this is not essential to the discussion here.

17 as well as a couple of other proposals not discussed in this thesis.
this context, clearly a wrong result. Under an “at least” meaning, \( \{d: \text{everybody else is at least } d\text{-tall}\} \) would be the singleton set of T, U and V's height (as one can easily see, that is the only height such that everybody but Q is at least that tall). (3.14) would then be predicted to be true in the given context, again not the desired outcome.

Further, Schwarzschild & Wilkinson point out that Quantifier Raising (QR) will not always work as a remedy in these situations; While assuming that in simple cases such as the one in (3.14) QR would be a valid option to predict the correct reading (and as Schwarzschild & Wilkinson point out one would still need to account for the absent narrow-scope reading), they present the following data (ibid:12ff):

(3.15) a. Bill did better than John predicted most of his students would do.

b. Most of John’s students are \( x \) such that Bill did better than John expected \( x \) would do.

Under QR (3.15a) would be predicted to be equivalent to (3.15b); yet in a context where John only predicted that most of his students would score within a particular range, yet makes no predictions regarding any individual student’s results, this would again be predicted to be undefined. As Schwarzschild & Wilkinson remark, equating the denotation of “than”-clauses with single points on a scale is insufficient to capture their meaning in cases where there’s several underlying degrees involved.

Rather, they suggest, “than”-clauses denote intervals of degrees, i.e. (connected) portions of a scale\(^{18}\). The above examples would in their framework then translate to (3.16) (informally).

(3.16) a. Q’s height is included in an interval \( I \) which is entirely above an interval \( K \) such that everybody else’s height is included in \( K \).

b. Bill’s score is included in an interval \( I \) which is entirely above an interval \( K \) such that John predicted that most of his students would score in \( K \).

\(^{18}\) Actually Schwarzschild & Wilkinson once refer to a “discontinuous interval” (ibid:28), i.e. the sum of several disjoint intervals. It is unclear to me whether they actually want to allow for this construct in their semantic theory; Approaches building on Schwarzschild & Wilkinson, especially Abrusán (2007) and Abrusán & Spector (2008, in progress), do not explicitly comment on this, but admitting sums of intervals into the semantics of degrees would from my understanding pose a major problem for their account, essentially invalidating their account of negative islands.
They then proceed to define operations necessary to compare intervals to each other, and in particular define a notion of measuring difference between intervals; in particular, the define:

\[(3.17) \ [I-K] = \text{the portion of the scale below } I \text{ and above } K, \text{ if } I \text{ is below } K; \text{ if } I \text{ is not wholly below } K, \ [I-K] = 0.\]

This in turn allows them to create a fairly sophisticated semantic system for comparatives that can account for a number of complex quantifiers in the “than”-clause that earlier theories had trouble accommodating. Schwarzschild & Wilkinson also apply their framework to equatives and numerals, making use of the notion of a contextually specified “unit interval”, two degrees being equal if they are both covered by a single such unit interval, i.e. if, for instance, Bill is 18 years and 4 months old, and John is 18 years and 8 months old, in a context where \([18 \text{ years}, 18 \text{ years and } 11 \text{ months}]\) is a unit interval, “Bill is as old as John is” would be predicted to be true.

Schwarzschild & Wilkinson also notice that there are both “exactly” and “at least” readings of equatives and numerals, as well as an “at most” reading in some cases, and remark that in their proposal it would be up to pragmatics to decide between those readings. I will return to this later.

An account of comparatives based on intervals also translates to the semantics of degree questions, although Schwarzschild & Wilkinson only hint at this. An approach to degree questions using interval semantics will be presented in chapter 4 however.

3.4 Maximal informativity in degree questions

Rullmann’s (1995) proposal was later revised in Beck & Rullmann (1996, 1999), basing exhaustivity in (degree and other) questions on maximal informativity rather than maximality. Their argument is in the same spirit as those of Heim (1994) or Dayal (1996), but specifically treats degree questions in addition to questions about individuals. As Beck & Rullmann note, Rullmann’s (1995) original account, while correctly establishing the connection between maximality and exhaustivity in simple cases, fails to account for cases such as (3.18).
(3.18) How many eggs are sufficient (to bake a cake)?

(ibid 1996:77, 1999:256)

In a case such as this, employing maximality would not yield the desired result, as they point out, for while there may be a certain minimal number of eggs that are sufficient to bake a cake (which the question is clearly asking for), there is no maximal number of eggs: If four eggs are sufficient, so are five eggs, and so are six eggs, etc. ad infinitum. Even if the number of eggs were contextually restricted to include an upper bound, Rullmann’s (1995) original account would still lead to a wrong prediction, namely that the question were asking for that upper bound.

Beck & Rullmann observe that this effect is due to a difference in the predicates in questions asking for a minimal answer and those in questions that require a maximal answer, and thus distinguish between upward and downward scalar predicates, defined as in (3.19), reformulated for easier reading in (3.20).

(3.19) a. A predicate P is upward scalar iff \( \forall n, m ((P(n) \& n \leq m) \rightarrow P(m)) \]

b. A predicate P is downward scalar iff \( \forall n, m ((P(n) \& m \leq n) \rightarrow P(m)) \]

(3.20) a. A Predicate P is upward scalar if and only if whenever it is true for a particular value \( x \) (degree), it is also true for any value greater than \( x \).

b. A Predicate P is downward scalar if and only if whenever it is true for a particular value \( x \) (degree), it is also true for any value smaller than \( x \).

In this view, predicates like “driving \( x \) fast”, ”having read \( x \) books” or “being able to jump \( x \) high” are all downward scalar: Having read four books entails having read three books and so on. On the other hand, “being sufficiently many eggs to bake a cake” and a few others are upward scalar.

Beck & Rullmann then provide a more principled account for the maximality and minimality effects by observing that in case of an upward scalar predicate, the minimal answer is in fact the maximally informative one. Noticing also that in the case of ques-

19 Unlike Rullmann (1999), Beck & Rullmann (1996, 1999) move to an “at least” meaning for degrees. Plain degree expressions would otherwise not be downward scalar. See also remark at end of section 3.2.
tions about individuals, maximality and maximal informativity still coincide, they propose that maximal informativity is the relevant domain in the semantics of questions.

They also note that some predicates are non-scalar, i.e. they do not allow for any inferences from a degree to smaller or larger degrees, illustrating this with examples such as (3.21).

(3.21) a. How many processors can Windows NT support?
    b. One, two and four.
(Beck & Rullmann 1999:258)

In other words, while upward scalar predicates simply “flip” the entailment pattern compared to downward scalar predicates, non-scalar questions get rid of them altogether - respectively, they exhibit entailment patterns akin to those of predicates over (plural) individuals in the above example\(^{20}\), or their semantics behave similarly to Schwarzschild & Wilkinson (2002)’s framework in other cases\(^{21,22}\).

### 3.5 Conclusion

We have looked at a number of theories on the semantics of degree questions that made different claims as to the meaning of degrees, how they combine to form sets of degrees, and how they license entailment patterns in predicates over the domain of degrees. While Schwarzschild & Wilkinson’s (2002) proposal was geared towards the semantics of comparisons, how their interval semantics might affect the structure of the H/K set of degree questions might already be tangible, and a proposal using Schwarzschild & Wilkinson’s approach to explain negative islands will be presented in the next chapter.

\(^{20}\) If Windows NT can support one, two and four processors, it follows that it can support one processor, that it can support two processors, etc.; However, note that this is not equivalent to saying that if it supports four processors, \(a, b, c\) and \(d\), it will also support any subset of those. See also section 4.4 of Beck & Rullmann (1999) on the latter.

\(^{21}\) Consider “How high can a plane fly?” which has a reading asking for both a minimum and a maximum, as Beck & Rullmann note. They do not get into examples like this one however.

\(^{22}\) They note that modals can create non-scalar predicates, but admit that their theory doesn't provide an account of why this happens (cf. Beck & Rullmann 1999:267). See also section 5.3.
Rullmann (1999) and Beck & Rullmann (1996, 1999) on the other hand focus on entailment patterns in degree predicates and have shown a link between maximality on the domain of degrees, and maximality of informativity on the domain of propositions. An amended version of their approach will also be presented that will tackle negative islands.

Finally, we have seen that whatever entailment patterns may be assumed for plain degree constructions, they vanish in some more complex cases, and degrees can be made to behave more like plural individuals, leading to corresponding entailment patterns in these cases.
Returning to the issue of negative islands, let us have a look at some of the relevant data again. Consider for instance (4.1) - (4.3).

(4.1)  
   a.  How fast did you drive?  
   b.  * How fast did you not drive?  

(4.2)  
   a.  Whom did you invite?  
   b.  Whom didn't you invite?  

(4.3)  
   a.  How many children do you have?  
   b.  * How many children do you not have?  

Note that (4.3b) is grammatical, although not under the intended meaning: It can be taken to ask for the number of children that the hearer doesn't have (out of some salient set of relevant children) as in (i), although this reading is hardly felicitous in this example. The meaning in (ii) is unavailable however:

(i)  What is the number of children \( n \) such that there are \( n \) children that you don't have?  
(ii)  * What is the number \( n \) such that it is not true that you have \( n \) children?  

In comparable positive questions both readings are available:

(i)  How many books do you want to buy?  
(ii)  What is the number \( n \) such that there are \( n \) books, each of which you want to buy?  
(iii)  What is the number \( n \) such that you want it to be the case that you buy \( n \) books?
The contrast in (4.1) and (4.3) also occurs in the corresponding comparative constructions:

(4.4)  a. John drove faster than Bill did.
       b. *John drove faster than Bill didn’t.

(4.5)  a. John has more kids than Bill does.
       b. *John has more kids than Bill doesn’t.

While syntactic accounts of these effects were already presented in chapter 1, as was noted there there are reasons to prefer a semantic account for these ungrammaticalities.

All of the following approaches will build on the assumption that certain kinds of contradictions can lead to ungrammaticality, and that questions presuppose the existence of a unique maximally informative true answer, as was proposed by Dayal (1996), hence referred to as the Maximal Informativity Principle (so called for instance by Abrusán & Spector 2008, in progress). For an elaboration on the former claim see chapter 6.

4.1 Maximality as a source of negative islands

While in chapter 3 Rullmann’s (1995) proposal was presented mainly from a point of view of exhaustivity, one of the main points of his thesis was to explain negative islands in degree constructions.

Consider how his maximality account would apply to a question such as (4.6):

(4.6)  a. What is the maximal degree \( d \), such that you drove \( d \)-fast?
       b. What is the maximal degree \( d \), such that you didn’t drive \( d \)-fast?

Clearly, there can never be such a maximal degree in the (b) case. Similar reasoning applies to the cases in (4.7):

(4.7)  a. What is the maximal number \( n \), such that you have \( n \) children?
       b. What is the maximal number \( n \), such that you don’t have \( n \) children?

As well as to the corresponding comparative constructions in (4.8), (4.9):
(4.8) a. John drove fast to a degree $d$ greater than the maximal degree $d'$ such that Bill drove $d'$-fast.
b. John drove fast to a degree $d$ greater than the maximal degree $d'$ such that Bill didn’t drive $d'$-fast.

(4.9) a. The number of children $n_\varepsilon$ that John has is greater than the maximal number $m_\varepsilon$ such that Bill has $m_\varepsilon$ children.
b. The number of children $n_\varepsilon$ that John has is greater than the maximal number $m_\varepsilon$ such that Bill doesn’t have $m_\varepsilon$ children.

Again, in all the (b) cases such a maximal degree / number does not exist: If John’s actual speed was $d$, then for any degree $d'$ greater than $d$, John didn’t drive $d'$-fast. Similarly, if the number of John’s kids is $n_\varepsilon$, then for any number $m_\varepsilon$ greater than $n_\varepsilon$, John doesn’t have $m_\varepsilon$ kids.

That all these cases are infelicitous follows directly from Rullmann’s (1995) maximality requirement as he points out: If, as in the above examples, the required maximal degree doesn’t exist, the entire utterance becomes bad.

However, while Rullmann’s (1995) proposal does account for the ungrammaticality of negative island cases, as Beck & Rullmann (1996, 1999) have pointed out, it does not make correct predictions for the meanings of certain cases involving upward-scalar predicates. Beck & Rullmann’s proposal, on the other hand, while preferable on other grounds does not account for negative islands at all. In their proposal, the above questions would be predicted to have the meanings as described in (4.10) and (4.11):

(4.10) a. * How fast didn’t you drive?
b. What is the maximally informative degree $d$, such that you didn’t drive $d$-fast?
c. What is the minimal degree $d$, such that you didn’t drive $d$-fast?

(4.11) a. * How many children do you not have?
b. What is the maximally informative number $n_\varepsilon$, such that you don’t have $n_\varepsilon$ children?
c. What is the minimal number $n_\varepsilon$, such that you don’t have $n_\varepsilon$ children?
Clearly, then, under this theory both of these questions would be predicted to be felicitous, and as asking for the minimal degree / number that will satisfy the predicate: Since “not having five children” entails “not having six children”, “not having seven children”, etc., in case one has four children, “I don’t have five children” would be the maximally informative answer to (4.11b).

And in a context where you drove just under 100 km/h, “I didn’t drive 100 km/h” would be the maximally true answer to (4.10b). Yet clearly neither of these questions can be taken to have such a meaning, in fact both are clearly ungrammatical.

This poses a bit of a conundrum: Either one opts to follow Rullmann’s (1995) original proposal, and have an explanation for negative island effects at the cost of getting wrong predictions in upward scalar cases, or one adopts Beck & Rullmann’s (1996, 1999) approach, in which case meanings involving minimality are established, yet one has to independently account for negative islands.

Additionally, not only does the straightforwardness with which the ungrammaticality of negative islands follows from Rullmann’s account suggest that there might be a link between negative islands and exhaustivity, as was pointed out in chapter 1 there are cases which no syntactic account as yet presented (or Rullmann’s 1995 proposal, for that matter) can account for. As Fox & Hackl (2006) have pointed out, certain modal constructions are able to obviate negative islands:

\[(4.12)\] a. How fast are we not allowed to drive?
   b. How fast are we required not to drive?

(Example from Abrusán & Spector 2008, in progress)

Both these cases are grammatical, yet none of the proposals hitherto presented account for this in any way, or even offer an obvious suggestion as to how they might be extended to cover these cases as well.

The following sections will thus present two proposals that take into account these cases. Again, both proposals are based on the assumption that failure to meet the pre-
supposition of having a unique maximally informative true answer can lead to un-grammaticality, although they implement this in rather different ways.

4.2 Density of Measurement: Fox & Hackl (2006)

Building on Rullmann’s (1995) original proposal and the observations made in Beck & Rullmann (1996, 1999), Fox & Hackl (2006) argue that maximal informativity is in fact responsible for negative island effects, although with an added twist, and also connect this to a number of other phenomena such as scalar implicatures.

Scalar Implicatures and “only”

They start by noticing that in sentences involving numerals or degrees, only some numerals respectively degree expressions trigger a scalar implicature while others don’t.

(4.13) a. John has three children.
   b. implies: John doesn’t have four children.

(4.14) a. John has more than three children.
   b. * implies: John doesn’t have more than four children.

(examples due to Fox & Hackl 2006:540ff)

In these cases, whenever the (4.13a) sentence is uttered, the speaker is taken to imply the sentence in (4.13b), yet no such implicature is seen from (4.14a) to (4.14b). As they note, this is directly related to the fact that in (4.13) the corresponding sentence with an “only” is acceptable, while in (4.14) it is not:

(4.15) a. John has very few children. He only has three_F (children).
   b. * John has very few children. He only has more than three_F (children).

(examples due to Fox & Hackl 2006:540ff)

They observe, building on a number of earlier proposals, that the scalar implicature of any sentence is essentially paraphrased by the same sentence with only and focus on

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25 This contrast was originally presented with focus / stress on the numeral three, as indicated by the subscript F. As this seems tangential to the discussion at hand however, I will not always explicitly mark focus.
the numeral. They label this the only implicature generalization (OIG): Any sentence, by default, licenses the implicature that the same sentence with only and focus on scalar items is true. The fact that this sentence would be ungrammatical in the case of (4.14), as in (4.15b), thus results in the absence of an implicature. To account for the ungrammaticality of (4.15b), then, Fox & Hackl turn to degree constructions. As they note, in a case such as (4.16) the unacceptability is easier to explain:

(4.16) a. John weighs 120 pounds.
   b. John weighs more than 120 pounds.
   c. * John only weighs more than 120$^P$ pounds.

Informally speaking, (4.16c) cannot be grammatical due to the density of the underlying domain of weights: (4.16c) presupposes that John's weight is greater than 120 pounds, say 121 pounds. However, John will then also weigh more than 120.5 pounds, contradictory to the claim that he only weigh more than 120 pounds. As the domain of weights is dense, no matter how little more than 120 pounds John actually weighs, there will always be some degree of weight in between 120 pounds and his actual weight, and John will also weigh more than that degree of weight, contradictory to the claim that he only weigh more than 120 pounds.

Fox & Hackl's then argue that the same line of reasoning also applies to the example in (4.13) - (4.14), and stipulate their central claim:

(4.17) The Universal Density of Measurements (UDM): measurement scales needed for natural language semantics are always dense. (Fox & Hackl 2006:542. Italics theirs.)

While the following line of reasoning might seem unintuitive in these cases, it does provide an account for the ungrammaticality of (4.15b) and the absence of an implicature in (4.14): “John only has more than three children” is ungrammatical because it presupposes that John has some amount of children larger than 3, for instance 3.1 (or 3.01, 3.00001, etc.). However, he then also has more than 3.05 (or 3.001, 3.000001, etc.) children, contradicting the claim that he only has more than 3.
Density and negative islands

Fox & Hackl then note that under the assumption of density of measurement, it is possible and rather straightforward to maintain Rullmann’s (1995) original account of negative islands arising due to failure to meet a maximality requirement, although replacing maximality with maximal informativity as suggested by Beck & Rullmann (1996, 1999). Consider again (4.18):

(4.18) * How much does John not weigh?

As per Beck & Rullmann this question is asking for the maximally informative degree $d$, such that John does not weigh $d$-much. As “John does not weigh $x$ pounds” implies “John does not weigh $y$ pounds” for any $y$ greater than $x$, it follows that the question is asking for the minimal such degree.

But by the same line of reasoning as before, such a degree cannot exist: If John’s actual weight is 120 pounds, then he does not weigh 121 pounds, or 120.1, or 120.0001, 120.000001,... pounds. No smallest degree $d$, such that John doesn’t weigh $d$-much, exists on a dense scale, hence the question is ungrammatical. And under the assumption of the UDM this account also extends to questions such as (4.19).

(4.19) * How many children do you not have?

Fox & Hackl's reasoning goes further: While in the examples considered thus far, the wh-phrase was always extracted out of the scope of negation, this should not be the deciding factor in determining grammaticality. Rather, failure to meet the requirement of having a maximally informative true answer should cause ungrammaticality. They thus explore cases where these two conditions don’t coincide.

Negative islands that are acceptable: modal obviation

In corroboration of their prediction that there be cases that formally are negative islands, yet do not lead to ungrammaticality, Fox & Hackl point out that in certain mo-

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26 The term “modal obviation” was introduced by Abrusán & Spector (2008, in progress)
dal constructions, a wh-phrase can move across negation without bringing about ungrammaticality:27

(4.20) a. How much are you sure that this vessel won’t weigh?
   b. How much radiation are we not allowed to expose our workers to?
   (Fox & Hackl 2006:551)

Yet as they point out, this only works if an existential modal scopes under negation, or if a universal modal scopes above it, but not the other way around:

(4.21) a. How much radiation is the company not allowed to expose its workers to?
   b. * How much food is the company not required to give its workers?

(4.22) a. How much radiation is the company required not to expose its workers to?
   b. * How much food is the company allowed not to give its workers?
   (Fox & Hackl 2006:552)

Fox & Hackl’s explain this quite easily:28 While there may not be a minimal degree such that the company in fact doesn’t expose its workers to that much radiation, there could well be a minimal dose of radiation it is not allowed to expose its workers to.

On the other hand, if there were a maximally informative answer to either of the (b) cases, i.e. x amount of food, such that the company is allowed not to give its workers that amount of food, then it would follow that the company is allowed to give its workers some amount less than x, x’ of food. But, again assuming density, there is an amount of food between x and x’ that the company is allowed not to give its workers. In other words, “x amount of food” would not have been the most informative answer, again a contradiction.

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27 They make the same point about modals & scalar implicatures and only, but their reasoning there is analogous to that about degree questions, and will thus not be repeated here.

28 Note that “not allowed to” = “required not to”, “allowed not to” = “not required to” (pointed out by Fox & Hackl 2006 as well).
As was mentioned earlier, but warrants repetition, any syntactic account of negative island effects would probably have great difficulty explaining the acceptability of the (a) examples above, making these data a key argument in favor of a semantic approach to negative islands.

**Negative islands that aren’t negative**

Fox & Hackl also mention the possibility that there be cases that are ungrammatical due to absence of a maximally informative answer, yet do not involve negation. They substantiate this prediction with the contrast in (4.23):

\[
(4.23) \begin{align*}
\text{a.} & \quad * \text{Before when did John arrive?} \\
\text{b.} & \quad \text{Before when do you have to arrive?}
\end{align*}
\]

(Fox & Hackl 2006:553)

Again, this is easily explained in their proposal, in the same fashion as the contrasts involving negative degree questions were accounted for: Whatever time John actually arrived at, given the assumption that the domain of time is dense, there will be no maximally informative answer to the question in (4.23a). In the (b) case however, there could well be an (earliest, hence maximally informative) time such that you have to arrive before that time.

As with the patterns observed with scalar implicatures, the same effects occur with questions involving intuitively discrete scales as do with degree questions, providing another argument for a unified account such as the UDM proposal:

\[
(4.24) \begin{align*}
\text{a.} & \quad * \text{How many children do you not have?} \\
\text{b.} & \quad \text{How many children are you not allowed to have?}^{29}
\end{align*}
\]

(cf. Fox & Hackl 2006:555)

**Maximal informativity, exhaustivity, and density**

As was already mentioned in section 2.4, exhaustivity in questions can be related to effects such as scalar implicatures, which were discussed at the beginning of the sec-

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29 Fox & Hackl name as a context to make this example more plausible: “If you live in China, how many children are you not allowed to have?”
tion on Fox & Hackl’ (2006) paper. While this was later generalized in Fox (2007) to include other domains as well, in the case of degree constructions this was already spelled out in Fox & Hackl (2006). They define a Max\textsubscript{inf}-operator that takes a predicate on degrees, and picks out the degree that gives the most informative true statement when combined with that predicate (i.e. for any predicate \(P\), it picks out the unique degree \(d\), such that \(P(d)\) is true, and for any other degree \(d'\), if \(P(d')\) is true, \(P(d')\) is entailed by \(P(d)\), if such a degree (respectively corresponding maximally informative statement) exists.

This then allows a straightforward definition of a number of concepts:

(4.25) **Exhaustivity:** The claim that a statement \(P(x)\) about a degree \(x\) is (strongly) exhaustive among a set of alternatives (e.g. all the true answers to a question) is true iff \(x = \text{Max}_{\text{inf}}(P)\)

(4.26) **Only:** “only \(P(x)\)” is true iff \(x = \text{Max}_{\text{inf}}(P)\), when that is defined.

(4.27) **Questions:** The denotation of a degree question is its H/K set iff. there is a degree \(x\) such that \(x = \text{Max}_{\text{inf}}(P)\). It is undefined otherwise.

(4.28) **the:** “the \(P(x)\)” = \(\text{Max}_{\text{inf}}(P)\), if defined.

(simplified from Fox & Hackl 2006:560)

The first two of these should be clear from the discussion earlier. (4.27) states that the denotation of a degree question is only defined if there is a maximally informative degree to answer it. (4.28) is used by Fox & Hackl to treat cases like “The amount of water such that [...].”

These definitions allow for a unified formal treatment of these effects. In Fox (2007) these definitions are translated mutatis mutandis to apply to predicates on any domain, moving from maximally informative degrees to maximally informative propositions. This generalized case is equivalent to the above in the case of degree constructions however.
Summary

Fox & Hackl’s proposal shows how in a semantic theory for degree questions following Beck & Rullmann’s (1996, 1999), density can be used to maintain an account of negative islands in degree questions similar to Rullmann’s (1995) original proposal, while still taking into account the modifications made to that theory by Beck & Rullmann to remedy the problems they pointed out, as discussed in section 3.4.

Their theory further extends to provide a unified account of the presence or absence of scalar implicatures in certain (modified) numerals and degree expressions, as well as the grammaticality of certain expressions and only.

Lastly, Fox & Hackl posit that density is also responsible for these effects in cases where relevant scales are intuitively discrete, e.g. “how many”-questions. That claim provides for a unified account of the identical patterns that these exhibit, but it is also a very strong claim about how the grammaticality respectively ungrammaticality of utterances is determined. This issue will be returned to in chapter 6.

4.3 Negation and intervals


In their proposal, the denotation of degree expressions does not correspond to single degrees, but rather to intervals of degrees. Consequently, degree questions are taken not to be asking for a single maximally informative degree, but to a maximally informative interval of degrees, as in (4.29):

(4.29) a. How fast are you driving?
   b. What is the (maximally informative, true) Interval $D$, such that your speed is included in $D$?

30 based on (parts of) Abrusán (2007), with the addition of the $\Pi$-Operator following Heim (2006).
Notice that in the above example, this reduces to the basic denotation “What is the speed $d$, such that you are driving exactly $d$-fast?” If one’s speed $d$ is included in an interval $D$ which is entirely included in a larger interval $D'$, i.e. $D \subseteq D'$, then it follows that one’s speed is also included in $D'$. In other words, the smaller the interval, the more informative it is. The most informative (degenerate) interval including one’s speed is thus the interval $[d, d]$, i.e. the singleton set $\{d\}$. There will, however, also be instances where this is not the case, for instance, in the case of questions involving modals.

(Revised) maximal informativity, intervals, and negation

Abrusán & Spector then posit that negation in a degree question in this framework will also lead to contradiction and ungrammaticality, although they have to adopt a slightly revised version of the Maximal Informativity Principle (MIP): In their theory, a question presupposes not only (i) that there necessarily be a maximally informative true answer, but also that (ii) this answer would not a priori be known as part of the common ground to be the maximally informative true answer, it would not _have_ to be the maximally informative true answer; that is, in terms of possible world semantics, it is presupposed that there be at least one possible world in which the actual maximally informative true answer would not be the maximally informative true answer (i.e. it is either not true in at least one possible world, or it doesn’t entail all other true answers in at least one possible world). Roughly speaking, this second claim states that questions don’t ask for information that is (necessarily) already known. This is obviously a derivation from Dayal’s (1996) original formulation, as reiterated by Fox & Hackl (2006), a fact on which I will remark later. Note however that the amendment they make will be crucial to their arguments.

Turning then to negative degree questions, Abrusán & Spector show that this requirement can never be met in simple cases:

(4.30) a. *How fast didn’t you drive?  
b. What is the maximally informative interval $D$, such that your speed is not included in $D$?

Assuming, for the moment, an “exactly” meaning of degrees. See following pages for a discussion of the $\Pi$-operator.
It may seem intuitive at first sight, considering that negation is akin to taking complement, that such an interval cannot exist: If one’s speed is $d$, any degree other than $d$ is a degree such that one is not driving exactly $d$-fast. But this set is the sum of two disjoint intervals: The one wholly below $d$, and the one wholly above $d$.

Yet as Abrusán & Spector point out, there is one case in which this line of reasoning does not hold, that in which one’s speed is exactly zero: In this case the entire scale above zero, i.e. the (open, unbounded) interval $(0, +\infty)$, would constitute the maximally informative answer to (4.30).

Cue their amended version of the Maximal Informativity Principle: Its first clause states that a question presupposes the existence of a maximally informative true answer. Yet this could, by the reasoning above, only be true if one’s speed were exactly zero, i.e. the only possible answer to 0a would be the interval $(0, +\infty)$. But this would violate the second clause of the MIP, requiring that the maximally informative answer not necessarily be the maximally informative answer. Therefore, a question such as (4.30a) can never satisfy this revised version of the Maximal Informativity Principle, and hence is ungrammatical.

**Modals and intervals**

Abrusán & Spector then go on to show that their proposal also accounts for the modal obviation as noted in Fox & Hackl (2006). Their argument is as follows:

\[(4.31)\]

a. How fast are we not allowed to drive?

b. For what interval $D$, it is not allowed that our speed be in $D$?

(Abrusán & Spector 2008, in progress)

Now while by the above reasoning there could not be a single interval such that our actual speed is not included in it, there could well be a situation where there is a maximal\(^{32}\) permitted speed\(^{33}\) $d$ (i.e. a speed limit); In such a context, the interval $(d, +\infty)$ would be the maximally informative answer to 0.

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\(^{32}\) Or a minimal speed one is required to drive, cf. footnote 12 in Abrusán & Spector (in progress).

\(^{33}\) Notice that in Fox & Hackl’s (2006) proposal a minimal disallowed speed was necessary to make their argument work. In Abrusán & Spector’s framework it is irrelevant whether one is allowed to drive at exactly the speed limit but no more, or only at less than the speed limit.
Conversely, Abrusán & Spector point out that a existential modal above negation does not obviate the negative island effect in their framework, again making the same prediction as Fox & Hackl (2006), as in the example in (4.32a).

(4.32) a.  * How fast are we allowed not to drive?34
b.  For what interval $D$, it is allowed that our speed is not in $D$?

(Abrusán & Spector 2008, in progress)

In this example their reasoning is a bit more complex:

Firstly, consider the case where there is no particular requirement on one’s speed. Then for any particular speed, not driving at that speed is allowed. Hence the maximally informative answer would have to be the entire scale of speeds, i.e. $[0, +\infty)$. Yet this would entail that it would be allowed for us not to have any speed at all (not even zero speed), respectively for our speed not to be in the domain of speeds, clearly a contradiction.35

Secondly, if there is a speed distinct from zero at which we are required to drive, there can again be no single maximally informative interval such that it is allowed that our speed is not in it: It is both allowed that our speed is not in the interval below the required speed, as well as in the one above it.

Lastly, if the required speed must be exactly zero, the interval $(0, +\infty)$ would be a maximally informative answer.

But it then follows, by parity of reasoning with the plain negative case, that $(0, +\infty)$ would necessarily be the maximally informative answer, violating the second clause of

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34 This example, amongst others, is sometimes prefixed with a hash sign in Abrusán & Spector (2008, in progress). For sake of consistency I will however stick to marking examples uniformly as ungrammatical.

35 Although (as a side note with no further consequences to the discussion at hand) I would like to point out that (4.32a) seems to have a marginally grammatical reading, difficult to conceive though it may be:

(i)  ?? How fast are we allowed “not-to-drive”? (resp. How fast are we allowed to “not-drive”?)
(ii)  ?? We are allowed to push our car at 10 km/h.

This seems to follow from Abrusán & Spector’s reasoning if one argues that in this situation there is no particular speed one is required to drive at. I think an example similar to the one above was briefly discussed in a class on the semantics of questions taught by Benjamin Spector in Vienna, March 2010, however I do not remember who brought it up.
the amended Maximal Informativity Principal. The amended MIP can thus never be satisfied for \( \emptyset \) either, again making the question ungrammatical.\(^{36}\)

Abrusán & Spector further argue that an interval-based reading can be shown to exist, and then extend their account to predict a few other cases not noticed by Fox & Hackl (2006). However as these don’t seem essential to the discussion, and the judgements in the latter cases appear to be relatively uncertain, I will not go into them. What is important to note is that while predictions might differ on some marginal cases, both Abrusán & Spector (2008, in progress) as well as Fox & Hackl (2006) make the correct predictions in those cases where a clear intuition about (un-)grammaticality is shared among speakers.

**The \( \Pi \)-Operator**

Abrusán & Spector note, however, that their approach doesn’t account for the standard, non-interval readings of certain questions, for instance (4.33).

\[(4.33) \text{ How fast are we allowed to drive?}\]

In their account as presented thus far, this can only ask for a maximally informative interval that our speed is allowed to be in. But, as Abrusán & Spector note, such an interval only exists if there is in fact only a single speed at which we are allowed to drive: If we are allowed to drive at any particular speed \( d \), it follows that it is allowed that our speed be included in any interval that contains \( d \), but not vice versa. Thus, any interval \( D \) such that “It is allowed that our speed be in \( D \)” entails all other true statements of the form “It is allowed that our speed be in \( D' \)” has to be a singleton set.

This, as they note, is a problem for their account, as it is just as plausible, if not more natural, to take (4.33) to ask for a maximum speed at which we are allowed to drive (as predicted by e.g. Rullmann 1995, Beck & Rullmann 1996, 1999, Fox & Hackl 2006).

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\(^{36}\) This argument crucially assumes that there is no upper bound on the domain of speeds. In case there were, i.e. if one were considering a case where the domain is of the form \([0, d)\) for some \( d \), both \([0, d]\) and \([0, d)\) would be possible maximally informative answers. Such a case, if it were to exist, would then presumably be predicted to be grammatical by Abrusán & Spector.
Following Heim (2006)\textsuperscript{37} they thus introduce an operator to mediate between single degrees as in the non-interval accounts, and intervals of degrees as in their account and that of Schwarzschild & Wilkinson (2002). This operator, dubbed \( \Pi \), short for \( p\cdot i \) or “point to interval”, in a nutshell, takes degree predicates such as “running \( d\)-fast” and turns them into a predicate on intervals of degrees, such as “one’s speed is included in \( D \)”. More precisely, it takes a degree predicate \( P(d) \), which it presupposes to have a maximum (i.e. it presupposes that there is a maximal degree \( d \) such that \( P(d) \) is true, but \( P \) is not true for any degree greater than that), and translates this into the set of all intervals of degrees that include that maximum.\textsuperscript{38}

A basic degree such as “\( d\)-fast” is then taken to have an “at least” meaning following Rullmann (1995) and others, which is turned into a degree meaning via \( \Pi \). For simple cases this reduces to their original proposal as they note:

\begin{align*}
(4.34) \quad & a. \quad \Pi (\text{John is (at least) } d\text{-tall}) \\
& b. \quad \text{i.e. any interval } D, \text{ such that the maximal degree } d, \text{ such that John is } d\text{-tall, is included in } D. \\
& c. \quad \text{i.e. any interval } D, \text{ such that John’s height is in } D.
\end{align*}

\begin{align*}
(4.35) \quad & a. \quad \text{How tall } \Pi \text{ (is Mary)?} \\
& b. \quad \text{i.e. What is the maximally informative interval } D \text{ such that the maximal degree } d \text{ to which Mary is tall is included in } D? \\
& c. \quad \text{i.e. What is the maximally informative interval } D \text{ such that Mary’s height is included in } D?
\end{align*}

The introduction of \( \Pi \) would then be rather redundant, if it weren’t for the possibility of \( \Pi \) taking wider scope in complex constructions; Abrusán & Spector thereby account for the missing reading of (4.33):

\begin{align*}
(4.36) \quad & a. \quad \text{How fast are we allowed to drive?} \\
& b. \quad \text{For what interval } D, \Pi \text{ (we are allowed to drive } d\text{-fast)?} \\
& c. \quad \text{For what interval } D, \text{ we are allowed to } \Pi \text{ (drive } d\text{-fast)?}
\end{align*}

\textsuperscript{37} In turn building on Schwarzschild (2004).

\textsuperscript{38} Technically, it takes two sets of degrees, the first a predicate that has a maximum and the second an interval, and returns the proposition that the maximum of the former is included in the latter.
As they note, the reading in (4.36c) with narrow-scope \( \Pi \) gives the reading originally predicted. However, having \( \Pi \) take wide scope as in (4.36b), we arrive at the “maximum-only” reading: This reading translates to “What is the maximally informative interval \( D \), such that the maximum degree \( d \), such that we are allowed to drive \( d \)-fast, is included in \( D \)?”, or equivalently “What is the maximum degree \( d \), such that we are allowed to drive \( d \)-fast?”.

An analogous argument is made regarding similar possible readings involving the universal modal “required to”, which I will not explicate here for sake of conciseness.

As Abrusán & Spector note, this amended theory still explains both plain negative islands as well as modal obviation: Consider again (4.30), here restated with \( \Pi \) taking both wide and narrow scope.

(4.31) a. * How fast didn't you drive?
   b. What is the maximally informative interval \( D \), such that it is not the case that \( \Pi \) (you drove \( d \)-fast)?
   c. What is the maximally informative interval \( D \), such that \( \Pi \) (you didn't drive \( d \)-fast)?

The (4.31b) case is as in their original proposal; the (4.31c) case is ungrammatical because \( \Pi \) is defined in terms of maximality, and there is no maximal degree \( d \) such that one didn't drive \( d \)-fast. In fact, as Abrusán & Spector note, all the readings with \( \Pi \) taking wide scope follow directly from Rullmann’s (1995) proposal due to the maximality requirement in the definition of \( \Pi \).

However, as they point out, there must be some restrictions on where \( \Pi \) can scope; they point out that “be supposed to drive \( d \)-fast” and “should drive \( d \)-fast” prefer a reading asking for both a minimum and a maximum (i.e. \( \Pi \) taking wide scope), whereas with “be required to drive \( d \)-fast” only a reading asking for a minimum seems to be available (i.e. \( \Pi \) taking narrow scope):

(4.32) a. How fast am I supposed to drive?
   preferred reading: unique degree or interval
   b. How fast should I drive?
   preferred reading: unique degree or interval
c. How fast am I required to drive?
   preferred reading: minimum required speed
   unavailable reading: # interval

They note that the same phenomena can be observed in corresponding comparatives:

\[(4.33)\] a. Jack drove faster than he was supposed to drive.
   (only) reading: higher-than-max
b. Jack drove faster than he should have driven.
   (only) reading: higher-than-max
c. Jack drove faster than he was required to drive.
   (only) reading: higher-than-min

This suggests that different \Pi\textsuperscript{-}scope might be in play in these constructions as well,\(^{39}\) although the details of which will not be spelled out here (and are only briefly outlined in Abrusán & Spector’s paper as well).

Abrusán & Spector leave the question as to what principles might govern this scope-taking behavior of the \Pi\ operator to future research however.

 Summary

Abrusán & Spector’s (2008, in progress) proposal builds on Schwarzschild & Wilkinson (2002), adopting a semantics for degrees based on intervals. This leads them to a rather straightforward explanation of negative islands except for a minor amendment to the Maximal Informativity Principal (but see the passage below on this subject). They then adopt a \Pi\ operator to account for additional readings unavailable under a plain interval-based account, and provide a “hybrid” account explaining negative islands as arising due to both their original interval-based account as well as due to violation of a maximality requirement, following Rullmann (1995).

\(^{39}\) However the mechanics of comparative constructions involving \Pi\ are beyond the scope of this thesis.
A short note on the excluded zero

As was explicated earlier, Abrusán & Spector (2008, in progress) have to amend the Maximal Informativity Principle in order for their account to work. I believe this makes their proposal unnecessary complex, and in my opinion less elegant than it could be. I would thus like to make a (tentative) suggestion to simplify their account.

As is generally accepted (see for instance Dayal 1996), questions like “Which people came to the party last night?” presuppose that somebody in fact did come to the party last night. (The question clearly sounds awkward in a context where nobody came.) I think this would easily generalize to degree questions.

It seems very plausible that a question like “How often does John work out?” carries the presupposition that John does work out. It sounds rather awkward in a context where it is known that John doesn’t do sports at all.

Similarly, a question like “How fast did you drive?” seems to presuppose that the hearer did in fact drive. Arguably driving implies movement, and hence a speed greater than zero. “I drove zero km/h” seems like a very strange thing to say to me.

With this generalization, then, the modification of the Maximal Informativity Principle as suggested by Abrusán & Spector isn’t necessary anymore for their arguments to hold, as the cases where one’s actual speed equals zero are ruled out on independent grounds.

This argument is rather sketchy I admit: In Dayal (1996) the presupposition a singular or plural wh-question has is explained due to the elements that go into the H/K-set (i.e. only singular or singular & plural individuals, while the empty set isn’t considered). In a similar fashion, one could assume that the zero point isn’t part of the H/K set of a degree question, thus explaining why such a question carries a presupposition.

But by this reasoning the unacceptability of (4.34) - or any other discrete case - isn’t accounted for straightforwardly.

(4.34) * How many children don’t you have?
In a situation where one actually has (exactly) one child, “I don’t have \([2, +\infty)\) children” would be a true answer, and entail all other true answers in the H/K denotation of the question in (4.34), if the zero point on the scale were excluded from the H/K set of (4.34).

One could perhaps assume that while (4.34) itself carries the presupposition that one has at least one child, the answer “I don’t have two or more children” doesn’t and thus doesn’t constitute the complete true answer. This line of reasoning isn’t entirely satisfactory I admit, but I’m not entirely sure how to resolve this problem. I will thus leave this as a rather tentative suggestion.

### 4.4 Conclusion

While Rullmann’s (1995) original account of negative had to be dismissed on independent grounds (pointed out by Beck & Rullmann 1996, 1999) two other semantic approaches to explaining negative islands in degree questions, both related to Rullmann (1995) respectively Beck & Rullmann (1996, 1999) in one way or another, were presented. Both build on the notion that negative islands are not rooted in syntax, but rather that they arise due to a question not (ever) being able to meet the presupposition that there be a unique maximally informative true answer to it.

Where they differ is in their assumptions about the nature and structure of the H/K denotation of degree questions:

Fox & Hackl (2006) assume a H/K set that consists of statements about singular degrees only. They assume degrees to have an “at least” meaning, thus making plain degree predicates upward monotone: If it is true that one drove \(x\)-fast, it is also true that one drove \(y\)-fast for any \(y\) smaller than \(x\).

In this framework there is thus a rather straightforward connection between entailment and position on a scale: Depending on the properties of a complex predicate, “more informative” means either “further up” or “further down” on the corresponding scale.
Scales being one-dimensional, this thus guarantees that there be at most one (hence unique) maximally informative element in the H/K set of a degree questions. Negative islands arise then if there is no such maximal element because to the underlying domain is (assumed to be) dense.

Abrusán & Spector (2008, in progress) on the other hand admit only intervals of degrees into the H/K set of a degree question. While in their proposal plain degrees also have an “at least” meaning, their Π operator is based on the maximal degree to which its argument is true, and consequently even in cases where the result is (minimally) a degraded singleton interval / set, the property that the Π operator returns is non-monotonic.

Thus, in their proposal, entailment corresponds to the “subset of”, “⊆”, relation on the domain of intervals of degrees. As obviously not any two degrees can be compared by that relation (they might be disjoint, or only partly overlap), not any two propositions in the H/K set of a degree question will be such that one is entailed by the other. This leads to the possibility that there be elements in the H/K set that are “locally” maximally informative, in that they entail all other true propositions in the H/K set which they can be compared to with regard to the “entailment” relation, yet which are not “globally” maximally informative, i.e. they do not entail all other true propositions in the H/K set. This is the case in plain negative degree questions, where both the interval below as well as the one above one’s actual speed / height / etc. entail “their halves” of the H/K set of the question, yet neither one entails the other.

Situations like this will thus generally be the source of negative islands in this account, coupled with the fact that these are assumed to coincide with the cases Rullmann’s (1995) original account already predicted to be ungrammatical, hence ruling these out in a wide-scope Π reading as well.

Notice that both proposals concern themselves with single degrees or intervals of degrees as abstract objects - neither of them is talking directly about linguistics expressions to describe those objects. It is not the point of either proposal that only plain degree expressions or “between x and y”-constructions are used to form the H/K set of

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40 Note that no plain degrees are considered for the H/K set in this framework. Single degrees are only considered as singleton sets / intervals.
a question, but rather the point(s) on a scale they represent. Crucially, however, while neither of them specify the linguistic form of the statements in the H/K set of a degree question, they do limit that set.

Also, neither of them assume modified numerals such as “more than three”, “three or more”, “no more than three”\(^\text{41}\) to take part in the generation of the H/K set of degree questions.

Lastly, notice one point that both approaches have in common, even though perhaps not obvious at first - While Fox & Hackl’s (2006) proposal superficially doesn’t seem to talk about sets of degrees at all, implicitly it does: In their theory, degree predicates are always monotone, that is, if \(P(d)\) is true for some \(d\), it is also true for all \(d’\) smaller (or greater) than \(d\) (depending on the predicate). But the set consisting of \(d\) and all degrees smaller (or greater) than \(d\) is \([0, d]\)\(^\text{42}\) (or \([d, +\infty)\)) - in other words, an interval. Thus, even in this approach there is an implicit link between entailment and the “subset of” relation: If \(d\) is smaller than (less informative than) \(d’\), then \([0, d]\) is a subset of \([0, d’]\). However since given monotonicity one endpoint of these intervals will always be the end of the scale, this reduces to the simpler case as outlined before.

\(^{41}\) See Nouwen (2008) for a very interesting discussion of “no more” expressions, providing an additional argument for density of measurement.

\(^{42}\) or \((-\infty, d]\) if a scale with no zero point / lower bound were involved.
Monotonicity and sets of degrees

We have seen in the preceding chapter that both an account based on single degrees and density, as well as one based on intervals of degrees and maximality are able to account for negative islands, yet they differ in some of their predictions, especially in modalized questions, and neither of them accounts for all the data hitherto presented.

5.1 Limitations of existing proposals

Modals and intervals

Consider again the following question, and the two answers given:

(5.1) a. How fast are we required to drive here?
    b. 80 km/h
    c. Between 80 and 130 km/h

Clearly, the answer in (b) is taken to mean that 80 km/h is the minimal speed at which it is allowed that we drive, but no slower. This is predicted straightforwardly under Fox & Hackl (2006), as well as under Abrusán & Spector (2008, in progress) with Π taking wide scope: If it is required that our speed be at least 80 km/h, then it also is also nec-
ecessary that our speed be at least 70, 60, ... km/h. 80 km/h is thus the maximally informative (Fox & Hackl 2006) respectively maximal (Abrusán & Spector 2008, in progress) speed $d$ such that it is required that we be at least $d$-fast.

However, the answer in (5.1c) is a bit more tricky, clearly meaning that it is required that our speed be between 80 and 130 km/h, but no more and no less. This follows directly from Abrusán & Spector (2008, in progress) with $\Pi$ taking narrow scope, but there is no way of deriving that meaning from Fox & Hackl (2006). If anything, under their proposal (5.1c) could be taken to mean that the speaker is uncertain about the minimal required speed and wants to convey reasonable upper and lower bounds for it, but in that framework there is no way of deriving a reading involving a maximal permitted speed from (5.1c).

This could be taken as an argument in favor of Abrusán & Spector’s (2008, in progress) approach, but consider also the following example:

$$(5.2) \begin{array}{ll}
\text{a.} & \text{How fast are we allowed to drive here?} \\
\text{b.} & \text{130 km/h} \\
\text{c.} & \text{Between 80 and 130 km/h} 
\end{array}$$

Similarly to the example with the universal modal, the answer in (b) clearly suggests that 130 km/h is the maximal allowed speed, again following directly from Fox & Hackl (2006) and from Abrusán & Spector (2008, in progress) with $\Pi$ taking wide scope.

As for the answer in (5.2c), there again seems to be a reading where it suggests uncertainty on the speaker’s part about the speed limit, 80 and 130 km/h being reasonable bounds according to the speaker’s beliefs. This again could arguably be derived from Fox & Hackl (2006),\footnote{As well as from Abrusán & Spector (2008, in progress) if $\Pi$ takes wide scope, of course.} if their theory is to make any sense if (5.2c) it all.

Abrusán & Spector (2008, in progress) on the other hand make a rather different prediction about an interval-based reading for (5.2a) (with narrow scope of $\Pi$), which one could expect to be underlying the answer in (5.2c). As they note, for any two intervals $I$ and $J$, if $I$ is a subset of $J$, it follows that “It is allowed that our speed be in $I$” entails
“It is allowed that our speed be in \( J \), but not vice versa, i.e. (informally speaking) intervals become more informative the smaller they get. It follows then, that a maximally informative interval could only be a “maximally small” interval - in other words a singleton set consisting of but a single speed. (cf. section 4.1 of Abrusán & Spector in progress)

But, as they point out themselves, this is not quite what one wants: It means that under an interval-based (narrow-scope II) reading of (5.2a), the only possible answer would be a single speed that is the only speed at which we are allowed to drive.\(^4\) This reading not only seems to be somewhat artificial,\(^4\) but it also means that Abrusán & Spector (2008, in progress) do not predict a sensible reading for (5.2c).

However, there arguably seems to be a reasonably natural reading of (5.2c) where it is taken to convey roughly the same information as the \((c)\) answer to (5.1a), namely that 80 km/h is the minimal and 130 km/h is the maximal permitted speed. But this reading is not predicted by either account. In short, while Abrusán & Spector (2008, in progress) are able to account for more readings than Fox & Hackl (2006) in modalized degree questions, they also can’t account for all of them.

**Non-monotone, non-connected properties**

As has been noted in the conclusion of chapter 4, both Fox & Hackl (2006) as well as Abrusán & Spector (2008, in progress) make very specific assumptions about the way predicates of degrees or sets of degrees entail each other, and consequently, what kind of sets of degrees take part in the formation of the H/K set of a question. In Fox & Hackl (2006), due to the direct link between informativity and position on a scale, only single degrees (respectively unbounded intervals as explicated at the very end of chapter 4) are used for the H/K set, whereas due to Abrusán & Spector’s (2008, in progress) reliance on intervals it is necessary that if a predicate is true for two degrees \( d \) and \( d’ \) it also be true for any degree in between. But consider the following example from Beck & Rullmann (1999):

\(^4\) In a context where there is more than one permitted speed, \( 0a \) under this reading is thus predicted to be infelicitous.

\(^4\) Abrusán & Spector mention in a footnote that it would be preferred to ask “How fast are we required to drive?” to ask for the single permitted speed.
(5.3)  a. How many processors can Windows NT support?
       b. One, two and four.

(ibid:258)

Clearly, neither proposal can adequately account for this data: Under Fox & Hackl (2006), mentioning one and two would presumably be rather redundant. From supporting (at least) four processors, it follows directly that Windows NT also supports (at least) one, two, and three processors. So not only would (5.3b) be predicted to be a rather odd answer to (5.3a), it would also get the wrong meaning, namely that Windows NT supports \emph{any} number of processors up to four, clearly not what (5.3b) is saying. Arguably, Fox & Hackl (2006) could even be taken to predict Windows NT to support zero processors from (5.3b), which is clearly an even worse inference.

Similarly, Abrusán & Spector (2008, in progress) would presumably either predict (5.3b) to also convey the information that Windows NT supports any number of processor between one and four (if 5.3b is assumed to mean the smallest interval that covers one, two, and four), or they would predict (5.3b) to be infelicitous. Again, neither of these predictions is what (5.3b) clearly is meant to say: That Windows NT can run (only) with exactly one, two or four processors, but not with any other number of processors (under the assumption that 5.3b is the maximally informative answer).

\section*{Minimality instead of maximality}

So far, the examples discussed in this section only pertained to the predictions made regarding interpretation, but not regarding grammaticality of the data involved. However, there are also some cases where the predictions made by Fox & Hackl (2006) and Abrusán & Spector (2008, in progress) differ. Consider the following (discussed by Fox & Hackl, but not by Abrusán & Spector):

(5.4)  a. * Before when did John arrive?
       b. Before when do you have to arrive?

Under Fox & Hackl (2006) this contrast is explained quite easily: There cannot be an earliest time before the actual time at which John arrived, making (5.4a) ungrammati-
cal, but there may well be a time such that you are required to arrive (strictly) before that time, thus making (5.4b) acceptable.

However, Abrusán & Spector (2008, in progress) would apparently make different predictions here:46

Under a wide-scope Π reading, (5.4a) would be ungrammatical because if John arrived before a given time, he also arrived before any time later than that, thus Π being undefined because there is no maximal time before which John arrived.

However, they would presumably also predict there to be a reading where Π scopes below “before”, making (5.4a) akin to “What is the maximally informative interval I, such that John arrived before that interval I’”. But then if John arrived at a time \( \tau \), the interval \((\tau, +\infty)\)47 would be the maximally true answer to (5.4a) under this reading. As a consequence, Abrusán & Spector (2008, in progress) would seem to predict (5.4a) to be grammatical with Π taking scope below require.48

As for the (5.4b) case, there seem to be three positions where Π can take scope: Highest scope above the modal, intermediate scope between the modal and “before”, and narrowest scope below “before”. The widest-scope reading should be unavailable as in the (5.4a) case, as should the intermediate-scope reading (again due to no maximal element being in the scope of Π), but the narrow-scope reading is again available.

This poses a bit of a conundrum: Either Π is able to scope below “before”, then both (5.4a) and (5.4b) are predicted to be grammatical, or for some reason it must scope above “before”, in which case both are predicted to be ungrammatical. But it seems that either way, Abrusán & Spector (2008, in progress) are unable to account for the contrast between the two.

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46 It could be argued that the example doesn’t actually apply to this discussion as it is about times, not degrees. However, the exact same reasoning could be with an example involving degrees, e.g. “Less than how much does John weigh?”

47 respectively \((\tau, \text{end-of-universe})\), depending on one’s convictions.

48 They didn’t discuss examples such as these, and did not elaborate on whether Π would be able to take scope in such a way. However, it seems from their remarks that they assume Π to be base-generated directly above the plain degree expression. Thus a reading as the one discussed would be expected unless some mechanism were stipulated that forced Π to move to scope at least directly above “before”.

5.2 Maximality versus maximal informativity in \( \Pi \)

As was shown in the preceding section, Abrusán & Spector’s (2008, in progress) proposal differs from Fox & Hackl (2006) in the predictions they make regarding the grammaticality of some examples.

Further, I would like to argue that to the extent that plain negative degree questions can be made acceptable, they can only ask for a minimal degree. Consider for instance the following context:

(5.5) A: So how rich exactly are you? Do you have, like, a million dollars?
    B: Yes, much more than that in fact.
    A: Like, even ten million dollars?
    B: Yes, easily.
    A: Even a hundred million?
    B: Yeah.
    A: Well, how much money do you not have?

Admittedly, the context is somewhat far-fetched, and A’s last utterance is still somewhat weird,\(^{49}\)\(^{50}\) but to the extent that it is felicitous, it is asking for the minimal amount of money (or a reasonable lower bound, perhaps) that B doesn’t possess. While neither approach would account for the relative acceptability of (5.5), Fox & Hackl would predict the correct minimum-reading for A’s question, by virtue of the minimal amount of money one doesn’t have being the maximally informative amount. Abrusán & Spector’s proposal would not capture this: They only account for interval and

\(^{49}\) It seems to be marginally acceptable with focus on “not”, as indicated.

\(^{50}\) Another interesting observation about this example is that the corresponding question in German seems to be somewhat felicitous in this context even with a “the hell” appended to the wh-phrase:

Wie viel Geld, zum Teufel, hast du nicht?
How much money, the hell, have you not?
“How much money, the hell, don’t you have?”

This seems to indicate that D-linking doesn’t play a role in obviating the negative island in the present case, following the assumption that “wh-the-hell” phrases are forcibly non-D-linked, following Pesetsky (1987). But see the discussion in footnote 5 of Abrusán (2007).
maximum readings, and don’t predict a “minimum” reading for (5.5) at all (regardless of grammaticality).

This seems to be a general limitation of this approach: It cannot accurately capture questions asking for the minimal degree to which a predicate is true. Note that questions with a universal modal are just an apparent exception to this: The minimal permitted speed is the maximal speed $d$ such that it is true the one is required to drive at least $d$-fast. Cases involving scale reversal, such as plain negation cases or the example with “before when?”, pose a problem for this account.

But this has already been noted by Beck & Rullmann (1996, 1999): The fact that Abrusán & Spector (2008, in progress) make wrong predictions in these cases is because the definition of the $\Pi$-operator is fundamentally building on a maximality requirement in the same way as Rullmann’s (1999) proposal.51 As Beck & Rullmann have shown, maximal informativity is crucial in making the correct predictions in these cases, not maximality.

Reconsider their crucial empirical argument:

\[(5.6)\]

\[\begin{align*}
  a. & \quad \text{How many eggs are sufficient to bake a cake?} \\
  b. & \quad * \text{How many eggs } \Pi \text{ (are sufficient to bake a cake)?} \\
  c. & \quad \text{How many eggs are sufficient } \Pi \text{ (to bake a cake)?}
\end{align*}\]

The wide-scope reading for $\Pi$ is ungrammatical for the same reasons as this question was predicted to be ungrammatical under Rullmann’s original proposal: There is no maximum number of eggs that are sufficient.

The narrow-scope reading for $\Pi$ translates to “For what interval $I$, it is sufficient that the number of eggs we have be in $I$?”. How this is interpreted depends on how “be sufficient” behaves:

If “Between $n$ and $m$, eggs are sufficient” means that a number of eggs between $n$ and $m$ is sufficient, by parity of reasoning with example (5.2), there can be no maximally

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51 The $\Pi$-operator was defined in Schwarzschild (2004) and Heim (2006) in connection with the semantics of comparatives, where maximality is obviously motivated by independent reasons. Yet I think even in comparatives wrong predictions would be made by this approach in examples similar to the ones described above for degree questions.
informative such interval: If it is sufficient that we have a number of eggs in \( I \), then it is sufficient that we have a number of eggs in \( J \) for any \( J \) that includes \( I \). Once again, then, there can only be a maximally informative interval if there is only a single number of eggs that is sufficient, and no other number of eggs will do to bake a cake, which in itself seems to be a contradiction.

If, on the other hand, “Between \( n \) and \( m \) eggs are sufficient” means that any number of eggs between \( n \) and \( m \) is sufficient, Abrusán & Spector would predict the narrow-scope \( \Pi \) reading of (5.6) to be grammatical. If the minimal number of eggs sufficient is \( n \), the answer would then be \([n, +\infty)\), which arguably seems to be what we want.

But then consider what happens if this example were embedded in a comparative, equative, or a definite:

\[
\text{(5.7) a. } \quad \ast \text{ I have more eggs than are sufficient to bake a cake.} \\
\text{b. } \quad \text{I have as many eggs as are sufficient to bake a cake.} \\
\text{c. } \quad \text{I have the number of eggs which is sufficient to bake a cake.}
\]

While (5.7a) is clearly ungrammatical, (b) and (c) seem significantly better, and to the extent that either of them is grammatical, both examples clearly refer to the minimal number of eggs sufficient to bake a cake, yet an approach using a \( \Pi \)-operator defined with maximality doesn’t seem to be able to account for this.\(^2\)

This seems to be a general observation: the arguments made by Beck & Rullmann (1996, 1999) about Rullmann’s (1995) original account seem to apply to some degree to Abrusán & Spector (2008, in progress) as well.

It hence seems worthwhile to extend the remedy they propose to Abrusán & Spector’s theory also, replacing maximality with maximal informativity in the definition of the \( \Pi \)-operator.\(^3\) Let’s call this modified \( \Pi \)-operator \( \Pi^* \). What would this change compared to their original proposal?

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\(^2\) One could add “The number of eggs I have is more than sufficient.” to the list, although it might be countered that the expression “more than sufficient” may have a different structure.

\(^3\) As Benjamin Spector (p.c.) told me, Danny Fox (p.c. to Benjamin Spector) proposed this as well.
Firstly, any readings with $\Pi^*$ taking narrow scope (directly above a plain degree) will not change in any way, as in this case (or any case where $\Pi^*$ scopes directly above an upward monotone property) maximality and maximal informativity coincide.

Secondly, in any readings with $\Pi^*$ taking wide scope, one would now get the exact same readings as predicted by Beck & Rullmann (1996, 1999) or Fox & Hackl (2006). This obviously gets the predictions regarding the “minimum” interpretation right, but as in Beck & Rullmann’s proposal an explanation of the sensitivity of these readings to negative island is, at least at first sight, lost.

However, I would like to point out that there is a straightforward way of getting around this problem, essentially unifying Abrusán & Spector’s (2008, in progress) proposal with that of Fox & Hackl (2006). If Universal Density of Measurement is assumed, the negative island effect in these readings would still be explained:

\begin{enumerate}
  \item How fast $\Pi^*$ (didn’t Jack drive)?
  \item What is the maximally informative interval $I$, such that the maximally informative (i.e. the minimal) speed $d$, such that Jack didn’t drive $d$-fast, is included in $I$?
\end{enumerate}

Obviously there can be no such interval, because there is no such minimal speed (any speed above Jack’s actual speed is a speed at which he didn’t drive, and assuming density, none of them is minimal). If all domains of measurement are assumed to be dense as in Fox & Hackl (2006), negative islands would hence be accounted for even with this modified version of Abrusán & Spector (2008, in progress).

In this modified proposal, the above facts are easily accounted for with $\Pi^*$ taking wide scope: (5.5), (5.6) and (5.7) are explained in exactly the same way as in Fox & Hackl (2006) then. (5.4b) also gets grammatical with $\Pi^*$ taking wide scope,\(^{54}\) again by the same explanation as in Fox & Hackl (2006).

It would appear, at this point, that one could discard Abrusán & Spector’s original proposal and move to an account involving $\Pi^*$ then, thus incorporating “the best of both worlds” so to speak.

\(^{54}\) This suggests that if one does posit a $\Pi$-operator, it should be prohibited from taking scope below “before” to account for the contrast between (5.4a) and (b).
Yet this is not the case. As it turns out, a definition of \( \Pi \) using maximal informativity instead of maximality is problematic in some cases involving comparatives. As Benjamin Spector (p.c.) points out, in comparatives there is no modal obviation:

\[(5.9)\]  
a. Jack drove faster than he was allowed to.  
b. *Jack drove faster than he wasn’t allowed to.  
c. *Jack drove faster than he was required not to.  
d. How fast was Jack not allowed to drive?

Clearly in the modified version of Abrusán & Spector’s proposal, this is not accounted for, both \((5.9b)\) and \((c)\) are predicted to be grammatical and have a “higher-than-min” interpretation, just as \((5.9d)\). Yet intuitively it is clear why these cases are ungrammatical: If the speed limit is, say, 130 km/h, than Jack isn’t allowed to drive at any higher speed than that. But obviously driving faster than \textit{any} speed greater than 130 km/h is impossible, short of driving at infinite speed perhaps.\(^{55}\)

It seems then that Fox & Hackl’s account or a modified version of Abrusán & Spector building on it may be well-suited to account for degree questions, but is unable to account for similar effects in comparatives. I will return to this issue at the end of this chapter.

\section*{5.3 Monotonicity and embedded exhaustivity}

Whether one opts for an account based on maximality and intervals, one based on maximal informativity and density, or a combined one as outlined in the preceding section, some issues remain. First and foremost, all of the hitherto discussed approaches do not in any way account for non-monotone properties such as \((5.3)\), here repeated as \((5.10)\).

\[(5.10)\]  
a. How many processors can Windows NT support?  
b. One, two and four.

\(^{55}\) Although even infinite speed is arguably a speed greater than 130 km/h, which one would eo ipso be required to drive faster than.
Clearly, in this case there is no logical entailment between the number of processors Windows NT can support, similar to a question about individuals such as (5.11).

(5.11)  
    a. Who is allowed to access the personnel files?  
    b. The CEO, the head of security, and the head of the HR department.

It seems, then, that the question in (5.10a) is more akin to something like (5.12a) or even (5.12b) than to a question about a monotone property of degrees.

(5.12)  
    a. What numbers of processors does Windows NT support?  
    b. What processor configurations does Windows NT support?

The question, then, is how a monotone property of degrees is taken to be a non-monotone property that seems to behave like a property of (plural) individuals.

One way of accounting for these non-monotone readings would be to posit that degree expressions themselves are ambiguous between an “at least” and an “exactly” reading. In such an approach, one would need to assume that for every degree expression there are in fact two lexical entries, one for each reading, e.g. “fast₁” meaning “at least $d$-fast”, and “fast₂” meaning “exactly $d$-fast”. However, I think this is not necessary.

Rather, I would like to propose, there is a much simpler explanation these ambiguities: Monotone properties of degrees can be turned into non-monotone properties under a modal by virtue of a covert exhaustivity operator scoping directly below the modal.

Clearly, in a simple degree question the “exactly” reading that an answer has stems from the fact that it is taken to be the maximally informative (i.e. exhaustive) answer:

(5.13)  
    a. How fast did you drive?  
    b. I drove (at least) $d$-fast.  
    c. exh (I drove at least $d$-fast) = I drove exactly $d$-fast.

I propose that the same mechanism is in play in modalized degree expressions.

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56 To the best of my knowledge, this hasn’t been pointed out in this form before. However, the proposal is clearly in the same spirit as a number of recent literature such as Chierchia, Fox & Spector (to appear).
Consider first simple modalized cases such as (5.14a) and (b).

\[(5.14)\]
\[
a. \quad \text{We’re allowed to drive } d\text{-fast.}
\]
\[
b. \quad \text{We’re required to drive } d\text{-fast.}
\]

Optionally placing a covert exh-operator below the modal yields two distinct readings for both statements:

\[(5.15)\]
\[
a. \quad \text{We’re allowed to drive (at least) } d\text{-fast.}
\]
\[
a’. \quad \text{We’re allowed to drive exactly } d\text{-fast.}
\]
\[
b. \quad \text{We’re required to drive (at least) } d\text{-fast.}
\]
\[
b’. \quad \text{We’re required to drive exactly } d\text{-fast.}
\]

The readings in (a) and (b) are the plain readings, with no exhaustivity assumed in the expression below the modal, the ones in (a’) and (b’) are assuming embedded exhaustivity. Consider now what happens if these statements themselves are taken to be maximally informative, e.g. as answers to a question about speed limits:

\[(5.16)\]
\[
a. \quad \text{exh (We’re allowed to drive at least } d\text{-fast.) = We’re allowed to drive } d\text{-fast but we’re not allowed to drive any faster.}
\]
\[
a’. \quad \text{exh (We’re allowed to drive exactly } d\text{-fast.) = We’re allowed to drive exactly } d\text{-fast.}
\]
\[
b. \quad \text{exh (We’re required to drive at least } d\text{-fast.) = We’re required to drive } d\text{-fast but we’re not required to drive any faster.}
\]
\[
b’. \quad \text{exh (We’re required to drive exactly } d\text{-fast.) = We’re required to drive exactly } d\text{-fast.}
\]

\[5.16a\) and \(b\) are the plain “maximal speed” and “minimal speed” readings predicted by any account following Rullmann (1995) or works building on it, such as Fox & Hackl (2006) or Abrusán & Spector (2008, in progress) with \(\Pi\) taking wide scope.

The readings in \((5.16a)\) and \((b)\) are somewhat different, and closer to the readings described in the first passage of section 5.1: \((b)\) means that we’re required to drive exactly at speed \(d\) (and are not allowed to drive at any other speed). Intuitively, this seems to be what a statement like “We’re required to drive exactly \(d\)-fast.” can be taken to mean.

\[\text{\textsuperscript{57} For the moment I’m ignoring pluralities of degrees and generalized quantifiers as alternatives for exh.}\]
The (a') case is a bit less satisfactory: In its present form it conveys the information that we’re allowed to drive exactly \(d\)-fast, but it doesn’t say anything at all about whether we’re allowed to drive at any other speed. That may be a rather nonsensical reading of “We’re allowed to drive \(d\)-fast”, although I think it can be taken to have this meaning.\(^{58}\)

Note that in the (a’) and (b’) cases applying \(\text{exh}\) again changes nothing: The statements of the form “We’re allowed / required to drive exactly \(d\)-fast.” (for any degree \(d\)) don’t entail each other, hence \(\text{exh}\) makes no difference here. (But see below.)

Before returning to this question, consider again the above example (5.3), but this time assume a covert \(\text{exh}\)-operator below the modal:

\[
\begin{align*}
(5.17) & \text{ a. Windows NT can exh (support four processors).} \\
& \text{ b. Windows NT can support (at least) four processors, and for any } n \text{ smaller than four, Windows NT can't support support } n \text{ processors.} \\
& \text{ c. Windows NT can support exactly four processors.}
\end{align*}
\]

Now this seems close to an accurate interpretation of (5.3), but obviously it would still be rather non-sensical to substitute “one, two and four” in the above example, as this would then seem to convey that Windows NT can support all three numbers of processors simultaneously.\(^{59}\) But observing that in similar cases involving plural individuals these properties seem to distribute to the atomic parts of the plural argument (“You’re allowed to have cake and ice cream” \(\rightarrow\) “You’re allowed to have cake” and “You’re allowed to have ice cream”), one could imagine (5.3b) to mean something like the following:

\[
\begin{align*}
(5.18) & \text{ a. Windows NT can support (at least) one, two and four processors.} \\
& \text{ b. Windows NT can exh (support one, two and four processors).} \\
& \text{ c. Windows NT can exh (support one processor), Windows NT can exh (support two processors), Windows NT can exh (support four processors).}
\end{align*}
\]

---

\(^{58}\) Imagine this being uttered on a road with no speed limit:

“We’re allowed to drive 100 km/h.”

“Duh. We’re allowed to drive as fast as we want!”

\(^{59}\) I can only imagine this making sense if one were talking about three different kinds of processors or something similar. Yet clearly this is not what is meant here.
d. Windows NT can support exactly one processor, and Windows NT can support exactly two processors, and Windows NT can support exactly four processors.

This, finally, seems to be an accurate meaning for (5.3a). Further, moving from single degrees to plural degrees, one can also refine the meanings of the modalized questions in (5.16a’) and (b’), realizing that in these cases the exhaustivity-operator has to take into account not only propositions about single degrees, but also about plural degrees. (5.16a’) and (b’) then get strengthened to the following:

\[(5.19)\]

a’. \(\text{exh (We’re allowed to drive exactly } d\text{-fast.)} = \text{We’re allowed to drive exactly } d\text{-fast and there is no other set of degrees } D \text{ such that } \{d\} \text{ is included in } D, \text{ and for any degree } d’ \text{ in } D, \text{ we are allowed to drive } d’\text{-fast.}\)

b’. \(\text{exh (We’re required to drive exactly } d\text{-fast.)} = \text{We’re required to drive exactly } d\text{-fast and there is no other set of degrees } D \text{ such that } \{d\} \text{ is included in } D, \text{ and for any degree } d’ \text{ in } D, \text{ we are required to drive } d’\text{-fast.}\)

Now the meaning in (a’) says that we’re required to drive exactly \(d\)-fast, and there is no other more informative set of speeds such that we’re allowed to drive at any of those speeds. In other words, it says that \(d\) is the only speed that we’re allowed to drive at.

This case would easily also accommodate a range of degrees that one is allowed to drive at, similar to (5.18).

Yet an account based on pluralities of degrees still cannot account for all the facts: Consider the following example (5.20).

\[(5.20)\]

a. How many processors does Windows NT require?

b. How fast are we required to drive?

Now the approach pursued in this section can’t properly deal with these. The question in (5.20a) can clearly be taken to ask for the same information as the earlier example, namely that Windows NT runs on one, two and four processor systems. Yet “one, two and four” would yield a rather nonsensical result here, namely that Windows NT requires all three of those to run.
Similarly, answering “between 80 and 100 km/h” to (5.20b) would mean that we’re required to drive at all speeds between 80 and 100 km/h simultaneously, clearly a rather absurd requirement.

Rather, both questions suggest a “Free Choice” scenario, in which Windows NT will work with either one, two or four processors, whichever one chooses, and similarly, that we’re required to drive at any particular speed of our choice, so long as it is between 80 and 100 km/h.

### 5.4 Generalized Quantifiers

Spector (2007, 2008) points out that this is a phenomenon observable in all modalized wh-questions. Consider the following example:

\[(5.21)\]

a. Which books must we read?

b. The French novels or the Russian novels.

c. Either we must read the French novels, or we must read the Russian novels, I don’t remember which.

d. We must read either the French novels or the Russian novels, whichever we prefer.

(Spector 2007:5)

Spector suggests that the (elided) answer in (5.21b) is ambiguous between the readings in (5.21c) and (5.21d). (5.21c) is a partial answer in the classic H/K or Groenendijk & Stokhof theories on interrogative semantics: It narrows down the possibilities as to what the answer to 0a to “We must read the French novels” and “We must read the Russian novels”.

(5.21d), however, poses a problem for these theories: As explicated earlier, the H/K denotation of a question “Which \(x\) P(\(x\))?” is made up of all the statements P(\(x\)), with \(x\) being any individual or sum of individuals that the wh-phrase ranges over. However,

---

\(^{60}\) Following Spector (2007, 2008) who observes this phenomenon for wh-questions, and Fox (2010).
“either the French novels or the Russian novels” isn’t such a (plural) individual. Rather, it is a Generalized Quantifier\(^{61}\) over individuals (books).

Spector thus suggests that wh-questions have another reading, in which they are type-risen from question over individuals to questions over Generalized Quantifiers. The question in (5.21a) could then be paraphrased as (5.22) in this reading.

\[
(5.22) \text{ For which Generalized Quantifier } G \text{ over books is it the case that we must read } G?^{62}
\]

As Fox (2010) suggests, this also translates to degree questions.\(^{63}\) Consider again the examples from before, in (5.23a') and (b') informally rephrased in a GQ reading.

\[
(5.23) \begin{align*}
\text{a.} & \quad \text{How fast are we allowed to drive?} \\
& \quad \text{a'.} \quad \text{For which Generalized Quantifier } G \text{ over speeds is it the case that we are allowed to drive at } G? \\
\text{b.} & \quad \text{How fast are we required to drive?} \\
& \quad \text{b'.} \quad \text{For which Generalized Quantifier } G \text{ over speeds is it the case that we are required to drive at } G?
\end{align*}
\]

Consider now what possible answers to (5.23a') and (b') could look like:

\[
(5.24) \begin{align*}
\text{a.} & \quad \text{We are allowed to drive at every (each) speed between 80 km/h and 100 km/h.} \\
\text{b.} & \quad \text{We are required to drive at some (any) speed between 80 km/h and 100 km/h.}
\end{align*}
\]

While the meaning of (5.24a) hasn’t changed substantially compared to the earlier approach based on pluralities of degrees, an account based on Generalized Quantifiers is

\(^{61}\) A full discussion of Generalized Quantifiers would be far beyond the scope of this thesis. If unfamiliar with the concept, assume that a GQ is a statement describing sets of individuals, e.g. “either the French novels or the Russian novels”, or “all the French novels and at least three Russian novels”, etc. For further information see for instance Barwise & Cooper (1981). Only a few simple cases of GQs will be relevant to the discussion at hand.

\(^{62}\) Spector further imposes the restriction that G be a monotone increasing quantifier.

\(^{63}\) Fox (2010) mainly shows that GQ readings are subject to the same negative island restrictions as plain degree readings. He doesn’t go into the exact interpretation such readings would receive, but suggests that they may account for the multiple readings in modalized degree questions.
also able to predict the correct meaning of the (b) case, namely that we are required to drive between 80 and 100 km/h, but which particular speed we choose in that range is up to us.

Notice the difference between the General Quantifier in (5.24a) and the one in (5.24b). In (5.24a) we are allowed to drive at all speeds in an interval, while in (5.24b) our obligations are fulfilled so long as we drive at any speed of our choosing within an interval. This difference is crucial:

(5.25) a. We are allowed to drive at a speed between 80 and 100 km/h.
b. We are required to drive at every speed between 80 and 100 km/h.

Here, the (5.25b) case is rather nonsensical: We can’t drive at more than one speed at one time, so being required to drive at all speeds between 80 and 100 km/h simultaneously is quite odd, and if at all, could only make sense in some rather strange contexts and meaning something along the lines of being required to drive at every speed in that interval at some point during e.g. a test run of a new car or similar.

The (5.25a) case is similarly rather odd: It seems to suggest that we’re allowed to pick any speed between 80 and 100 km/h and drive at that speed. But we’re only allowed to pick one speed. Again, as one can only be driving at one speed anyways, this is rather odd.64 (This might be somewhat better in a context where a group of several is required to drive at any speed in an interval, so long as they all drive at the same speed.)

It should now also be clear why the interval-based approach had trouble dealing with the “allowed to”-cases, and why the pluralities of degrees-based approach had trouble with the “required to”-cases: The former because it assigned these cases a meaning along the lines of “For which interval I are we allowed to drive at a speed in I”,65 the latter because it assigned the problematic cases one like “For which set of speeds D are we required to drive at every speed in D”. As just discussed, neither of these readings

---

64 The oddness of both cases stems from the fact that one can only drive at one speed. Similar examples with plural individuals work fine, as in “You’re allowed to have one of the desserts on the menu” vs. “You’re allowed to have all of the deserts on the menu”, and “You’re required to read one of the books on the reading list” vs. “You’re required to read all of the books on the reading list”.

65 Further complicated by the fact that while in these cases “(any) one degree in I” could be the maximally informative GQ, in Abrusán & Spector’s account only a singleton set can be the maximally informative interval.
makes much sense. However, unlike an account building on Generalized Quantifiers, neither the interval-based account nor one based on pluralities of degrees has the flexibility of allowing both for a “one of these” and an “all of these”-construction.

Lastly, notice that the meanings predicted for the modalized examples in (5.24) get further refined if they are taken to be maximally informative amongst other possible answers based in Generalized Quantifiers (as opposed to answers based on degrees or pluralities of degrees):

(5.26) a. exh (We are allowed to drive at every (each) speed between 80 km/h and 100 km/h.)

b. exh (We are required to drive at some (any) speed between 80 km/h and 100 km/h.)

If one restricts the discussion to Generalized Quantifiers of the form “every x in a particular set” respectively “some x in a particular set”, these work out to the following:

(5.27) a. We are allowed to drive at every speed in I, and there is no more informative true statement of the form “We are allowed to drive at every speed in J”.

b. We are required to drive at some speed in I, and there is no more informative true statement of the form “We are required to drive at some speed in J”.

This is a welcome result: (5.27a) states that there is no set of speeds J larger than (i.e. superset of) I, such that we are allowed to drive at every speed in J, i.e. there is not speed at which we are allowed to drive other than the ones in I.

(5.27b) states that there is no set of speeds J smaller than (i.e. subset of) I, such that we are required to drive at some speed in J, in other words, every speed in I is such that we are allowed to drive at that speed, i.e. we are really free to choose any speed in I that pleases us.

As a side note, Abrusán & Spector (2008, in progress) argue in footnote 30 that whenever only a distinct set of options (of any kind) is allowed, and nothing more, then there is a tendency to prefer a universal modal over an existential one, e.g. if it is
known that only a particular speed (or set of speeds) is allowed, one would rather ask about the required speed than the allowed one. I think it follows from the reasoning above that this is not necessarily true. Not only does “How fast are we allowed to drive?” get a much more complex reading under the GQ approach than under one based on intervals, but even in the scenario of only a single allowed speed (as presupposed in an interval-based approach) there is a distinct difference in meaning between an existential and a universal modal, even in the fully exhaustified meanings as in (5.27a) and (b): “We are only allowed to drive exactly $d$-fast” doesn’t say anything about whether we are required to drive at all, whether from “We are (only) required to drive exactly $d$-fast” it seems to follow that we are in fact required to drive. In an account based on Generalized Quantifiers, this follows directly.\textsuperscript{66, 67}

A full discussion of Generalized Quantifiers would be beyond the scope of this thesis. What is important to note is that if one assumes that degree questions have a GQ-reading (or even a “plural degrees” / “plurality of degrees” / “set of degrees”-reading), one has to show that this reading is also subject to the negative island effects discussed earlier. Fox (2010) does this, albeit for the general case (i.e. not just limiting the proof to degree questions), and therefore it might not be practical to repeat his proof in full in a discussion about degree questions.

Crucial to the reasoning of his proof, Fox makes a rather substantial amendment to the Maximal Informativity Principle. Due to the much more complex domain of Generalized Quantifiers, it isn't sufficient for a question $W$ to presuppose that it has a maximally informative true answer. Instead, it must be possible for every answer (i.e. every statement in the H/K denotation of a question) to be the maximally informative true answer to $W$, i.e. there must be no possible answer in the H/K set of $W$ that is a

\textsuperscript{66} Assuming that the propositions are exhaustified with regard to alternative Generalized Quantifiers, not a broader set of alternative propositions (e.g. being allowed not to drive at all).

\textsuperscript{67} Similar reasoning applies to their second example: If Jack is required to read books A, B and C and is not allowed to read any other books, it would indeed be odd to ask “Which books is Jack allowed to read?”. However, if he’s not required to read them, but rather he has the choices of reading A, B and C and nothing else, or reading nothing at all, then it would be preferred to ask about which books he is allowed to read, rather than which books he is required to read.
priori ruled out as the potential maximally informative true answer to \( W \) by logical reasoning alone, irrespective of what the facts of the universe potentially might be.\(^\text{68}\)

For a formal proof that the Generalized Quantifier-reading of questions is subject to the same negative island effects (as well as the same types of modal obviation) as the plain reading of degree questions I refer the reader to Fox (2010).

### 5.5 Conclusion

After observing a number of cases that neither Fox & Hackl (2006) nor Abrusán & Spector (2008, in progress) could fully account for, or where their predictions differed, a number of alternatives were investigated.

An account based on Abrusán & Spector, but with a modified \( \Pi^* \)-operator based on maximal informativity rather than maximality, was proposed that essentially united Abrusán & Spector’s proposal with that of Beck & Rullmann and Fox & Hackl. This approach accounted for a number of phenomena with degree questions, but lost some predictions with comparatives compared to Abrusán & Spector’s original proposal.

Then, I argued that an optional covert exh-operator directly below a modal can explain the ambiguity between monotone and non-monotone readings that these predicates show.

An account using pluralities of degrees was considered, but shown not to account for all readings in modalized degree questions.

Finally, an account using Generalized Quantifiers over degrees, as suggested by Spector (2007, 2008) and Fox (2010), was investigated. This approach seems to account for most, if not all, of the data hitherto reviewed. (Although there are still some loose ends, as Fox 2010 points out.)

\(^{68}\) One thing is important to point out I think: This modified version of the MIP does not say that a question be required to have a maximally informative true answer in any situation / context. It very much does allow for a question not be answerable in a given context. Its requirement is quite the reverse in fact: For every potential answer in the H/K set of a question, there must be a situation in which it is the maximally informative true answer.
As a final remark, it seems that the GQ-approach seems to account for the lack of modal obviation in comparatives that Benjamin Spector (p.c.) pointed out, as mentioned in the relevant section. Assuming that “-er than” itself refers to maximality, it seems reasonable that in a GQ-reading of the embedded clause the maximal of any element of any witness set of the GQ is relevant. As a consequence, it is clear why (5.28) is unacceptable: There simply is no such maximal element.

(5.28) * He was driving faster than he wasn’t allowed to.

But it seems to me that in the right context a similar thing could be felicitous, if only marginally. Imagine due to security concerns airplanes are not allowed to fly below 1200 m in a certain area. Then I think the following should be acceptable:

(5.29) The pilot claimed he was in fact flying higher than he was not allowed to.

This account also seems to be in line with Schwarzschild & Wilkinson’s (2002) proposal for the semantics of comparisons: No obvious reason comes to mind why their framework shouldn’t be able to deal with an interval covering (all witness sets of) a Generalized Quantifier.
Contradiction and grammaticality

One of the central assumptions of the preceding chapters was that contradiction respectively inability to meet a presupposition can lead to ungrammaticality. In the case of degree questions this has been shown to be the case in negative islands by both Fox & Hackl (2006) and Abrusán & Spector (2008, in progress): A negative degree question, unless placed under a universal modal, can never have a maximally informative true answer, regardless of what the actual facts of the world are. Yet while this approach has been shown to have merits it raises the question why some contradictions are ungrammatical, while others are not. As Fox & Hackl (2006) and others note, sentences such as the ones in (6.1) are perfectly acceptable, even though they are clearly contradictory.

(6.1)  
   a. This table is both red and not red.  
   b. He left and didn’t leave.  
   c. I have a female father.  
   (examples following Fox & Hackl 2006)

Clearly, in order for the arguments presented in this thesis to be valid, one also has to provide an argument for what accounts for this contrast.
I will briefly sketch the proposals that have been made in this area, even though I won’t be able to thoroughly discuss the subject here.

6.1 L–analyticity

Gajewski (2002) proposes that the difference between contradictions that result in perfectly acceptable sentences, and those that lead to ungrammaticality, is that the latter arise from their logical structure alone, regardless of what the non-logical constituents of a sentence are. That is, in his proposal for the process of determining whether a sentence is ungrammatical due to contradiction, every non-logical element is replaced with a variable of respective type, and the resulting logical structure is evaluated.

That is, roughly speaking, in a sentence like (6.1b), the structure that is used to determine grammaticality is akin to (6.2).

\[
(6.2) \quad \forall v_1,\text{DP} \quad \forall v_2,\text{VP} \quad \text{and not} \quad \forall v_3,\text{VP}.
\]

Clearly, from the logical skeleton in (6.2) alone no contradiction follows.

Gajewski also argues that sentences that are always true due to their logical form alone are similarly ungrammatical, examples including expressions such as (existential) “There is every student.”

Gajewski calls these expression that are necessarily true or false by virtue of their logical skeleton alone “L-analytic”, to distinguish them from other analytic expressions, such as the contradictions in (6.1).

---

69 See Gajewski (2002) for an exact definition of what he considers logical and non-logical elements.

70 Somewhat simplified: In Gajewski’s original proposal every variable is of appropriate semantic type, but semantic types were intentionally left out of this thesis as they were not central to the arguments about degree questions that were presented.

71 Although these kinds of examples are of no relevance to the discussion at hand.
6.2 The Deductive System

Fox (2000) argued that while the cognitive systems that handle grammar (syntax) and those that provide logical reasoning (semantics & pragmatics) are independent entities, there is also evidence of a system in between, dubbed the “Deductive System”, that provides a facility of formal reasoning that is able to interact with syntax.72

Fox & Hackl (2006) take this further and argue, based on Gajewski (2002), that the formal reasoning the Deductive System is capable of is blind to lexical information.

As a consequence, its formal calculus is incomplete: It isn’t able to derive all the consequences from an utterance that the semantic & pragmatic system is able to deduct. For instance, it doesn’t rule out sentences like the ones in (6.2), even though they are logical contradictions.

But further, they argue, it is also not sound: Their argument crucially relies upon the notion that for the purpose of determining a sentence’s grammaticality, certain kinds of lexical information are not considered. In particular, their claim that all scales are formally dense assumes that in the case of scales that are intuitively discrete, this piece of information is not available to the Deductive System.73

Fox & Hackl also point out that the assumption of a Deductive System as above also explains why some expressions that are predicted to be ungrammatical given density of measurement are in fact unacceptable, while some are not:

(6.3) a. *Before when did John arrive?
b. The earliest time after ten.
c. The \( v_{1,AP} <est v_{2,NP} \) after \( v_{3,NP} \)?

---

72 Fox’s original argument was related to certain scope operations. See Fox (2000:66-74) for further details.

73 Notice that completeness and soundness are also two fundamental concepts in mathematical logic. It seems to me that the kind of formal reasoning of the Deductive System Fox & Hackl propose, as opposed to semantical / pragmational reasoning, is similar to the distinction between syntactic entailment and semantic entailment in mathematical logic.
While, as they point out, the ungrammaticality of (6.3a) can be derived as long as the preposition “before” is part of the vocabulary accessible to the Deductive System, in the case of (6.3b) it is reasonable to assume that an adjective like “early” would not be visible to the Deductive System. As a consequence, (6.3c) can’t be ruled out if one considers that other adjectives such as “opportune” or similar could be in ν₁’s place.

Lastly, Fox & Hackl argue that the Deductive System could plausibly also be unable to conclude that the domain denoted by any DP was limited to a finite set of individuals, but instead would have to assume that the set denoted by any DP was infinite. This could be taken to explain why negative degree questions such as (6.4b) are relatively acceptable.

(6.4) a. * How many assignments did John not hand in?
   b. ? How many assignments did no students hand in?

(ibid:576)

If the domain of students were infinite, it would be possible that, given density, that there be a number of assignments n such that for every number even infinitesimally smaller, m, a student handed in m assignments. “No student handed in n assignments” would then be the maximally informative true answer to (6.4b).

6.3 Conclusion

Both Abrusán & Spector’s (2008, in progress) interval-based account of negative islands as well as Fox & Hackl’s (2006) proposal share the common assumption that certain kinds of contradictions can lead to ungrammaticality. Gajewski (2002) proposed a way of distinguishing these contradictions from those that are found in perfectly grammatical statements.

Both Abrusán & Spector and Fox & Hackl build on Gajewski, but their proposals differ in the assumptions they make as to what information is available to the cognitive facility determining grammaticality: Fox & Hackl’s proposal crucially relies on the assumption that density of the underlying scales is always assumed in determining acceptability, whereas Abrusán & Spector don’t make such strong claims.
Notice that Abrusán & Spector also rely on the assumption that lexical information is not taken into account to rule out ungrammatical sentences. Otherwise they would have to predict a question like “How many children does a childless couple have?” to be ungrammatical, which it clearly isn’t.

The crucial difference between the two proposals in this regards thus seems to be whether the Deductive System (or any other entity responsible for ruling out logical contradictions) is able to distinguish between “how many” and “how much”-questions, broadly speaking.\textsuperscript{74}

The issues presented in this chapter have only been discussed in the literature fairly recently. Many questions remain open and most conclusions are tentative. But whatever conclusions will be arrived at in this area, it is obvious that the question of how contradictions are able to affect grammaticality is central to our understanding of how syntax and semantics interact.

\textsuperscript{74} I suppose if there were a language that didn't mark the difference between “how many” and “how much” in any way (i.e. morphologically, syntactically, etc.), this would be an argument in favor of the claims Fox & Hackl make about the Deductive System. I don’t know if any such language exists.
Conclusions

Taking some general observations about the semantics of questions as a starting point, this thesis presented a number of works on the semantics of degree expressions and then introduced negative island effects.

The main point of the discussion was a comparison of Fox & Hackl (2006) and Abrusán & Spector (2008, in progress). I have pointed out a number of cases where these two proposals differed in their predictions, or which were outside their limits altogether.

I then investigated how both accounts could be unified using a slightly modified version of the Π-operator, but also that such an approach still couldn’t account for all the problems that were observed.

Further, I proposed that monotone degree properties become non-monotone under some modal operators by virtue of an optional covert exhaustivity operator directly below the modal.

Lastly, an approach was outlined that assumed that degree questions had a General-ized Quantifier-reading in addition to a plain degree reading, as proposed by Fox
following Spector (2007, 2008). This GQ-approach to higher-type readings of degree questions seems to be the latest development in this area and appears to account for all the facts presented in this thesis. As was mentioned, Fox (2010) further showed that a GQ-reading is subject to the same negative island constraints that a basic degree reading is.

It follows then that this discussion will leave off at this point: For plain degree readings, Fox & Hackl (2006) provide an account of negative island effects, while for higher-type readings a GQ-based account provides both correct predictions regarding available readings in modalized cases, while its susceptibility to negative islands has been accounted for by Fox (2010). As Fox (2010) notes, there are a few open questions surrounding the GQ approach, and further research in either that direction or towards an interval-based reading is thus warranted.

Finally, it was briefly outlined that at the heart of the debate is also the question of how syntax and semantics interact, and how grammaticality of an utterance may also be determined by an apparatus of restricted formal reasoning. Research in this area is fairly recent and at times conjectural, but given the rather fundamental nature of the questions about the workings of the linguistic system that it poses, seems to have relevance beyond the field of formal semantics.

Therefore, I conclude hoping that the present thesis make the issues at hand accessible to a broad linguistic audience.


Abrusán, M, and B Spector. in progress. A Semantics for Degree Questions Based on Intervals: Negative Islands and their Obviation.


Fox, D. 2010. Negative Islands and Maximization Failure: Handout to a talk given at MIT Linguistics Colloquium.


Schwarzschild, R. 2004. Scope-Splitting in the Comparative: Handout to a talk given at MIT Linguistics Colloquium.


Abstract (english)

The present thesis is an attempt to give an overview over recent literature in the field of the semantics of degree constructions, and in particular over semantic accounts of negative island effects.

To do this, I will first review some theories in the semantics of questions in general, particularly those developed by Hamblin (1958, 1973), Karttunen (1977), Groenendijk & Stokhof (1982) and Heim (1994). I will particularly focus on the relationship between domains of wh-phrases, sets of propositions and sets of possible worlds, and how these play together in the semantics of questions. Building on these explication I will arrive at the concept of exhaustivity and maximal informativity.

Next, I will present some theories on the semantics of degree constructions, and will try to show how the concepts introduced in the preceding chapter are connected to these.

I will then turn to two recent semantic approaches to negative islands, those presented in Fox & Hackl (2006) and Abrusán & Spector (2008, in progress). Some remarks will be made on the implications that these theories have on the structure of the linguistic system.

Finally, a number of further approaches will be considered, with an approach based on Generalized Quantifiers as proposed by Fox (2010) and Spector (2007, 2008) being the conclusion of the debate.

A brief outline of the issues regarding the interface between syntax and semantics that underly the current debate will be given at the end of this thesis.
Abstract (deutsch)

Die vorliegende Arbeit ist ein Versuch einen Überblick über neuere Literatur auf dem Gebiet der Semantik von Gradkonstruktionen zu geben, und insbesondere auf semantische Theorien zu Negativinseln einzugehen.


Anschließend werden einige Theorien zur Semantik von Gradkonstruktionen behandelt, und wie diese mit den Konzepten aus dem vorherigen Kapitel zusammenhängen.


Am Ende der Arbeit wird kurz auf die Fragestellungen im Zusammenhang mit dem Verhältnis von Syntax und Semantik eingegangen, die die behandelte Debatte aufwirft.
CV (english)

Education

Sir Karl Popper-Schule, Vienna — 2001 - 2005

University of Vienna — 2005 - 2007 and 2009 - 2010
Mag.phil. programme in Linguistics.

University of Vienna — 2005 - 2007 and 2009 - 2010
Mag.rer.nat programme in Mathematics; discontinued due to switch to Cambridge.

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Work & Volunteer Experience

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JHC, Melbourne, Australia — 02/2008 - 01/2009
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